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MATHEMATICAL MODELLING OF THE HEAT TRANSFER PHENOMENA IN VARIABLE SECTION FUSIBLE ELEMENTS

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Summary

The paper presents a general heating equation, in overload regime, of the fusible elements designed for current-limiting electric fuses, fusible elements made up of silver tapes with rectangular constristions. An original method for mathematically solving this partial differential equation of a parabolic type and, finally, a comparison between the computation results and the experimental ones obtained when testing some fuse-links for power transformer protection, are presented.

List of symbols

- γ - specific mass, kg/m^3
- c - specific heat, Ws/(kg.^{o}C)
- λ - thermal conductivity coefficient, W/(m.^{o}C)
- K - side surface heat transfer coefficient, W/(m^2.^{o}C)
- ρ - fusible element resistivity, Ωm
- J - electric current resistivity, A/m^2
- l_x - conductor peripheric length (perimeter), m
- A_x - conductor cross-section area, m^2
- θ_a - ambient temperature, ^{o}C
- θ - fusible element temperature, ^{o}C
- ρ_o - fusible element resistivity at 0^{o}C, Ωm
- α_o - resistivity variation coefficient, function of temperature, at 0 ^{o}C, 1/^{o}C
- g - fusible tape thickness, m
- a - fusible tape minimum half-width, m
- b - fusible tape maximum half-width, m
- d - perforation half-length, m
- i - current instantaneous value, passing through the fusible element, A
- I - current effective value, A
- I_o - current effective value on a fusible element in overload regime, A
- θ_o - fusible element initial temperature, ^{o}C
- θ_f - fusible element fusion temperature, ^{o}C
- t_f - time necessary for the fusible element temperature to reach its fusion temperature, θ_f, in the x = o point, s
- θ_{max} - maximum temperature the fusible element can reach, ^{o}C
- T - time constant, s
- n - number of the interval divisions [o, d]
- Δx - discretization step
- N - system equation number obtained through discretization.

1. Introduction

High-voltage current-limiting fuse-links are widely used for short-circuit and overload protection of the electrical circuits that contain motors, power capacitors or high-power distribution transformers.
The complex nature of the heat flux inside the current-limiting electric fuses makes impossible the direct analysis of the phenomena, using classical techniques. The specialized literature briefly presents various computation methods, which mainly make use of finite difference techniques, for establishing the behaviour of the current-limiting fuses under overload and short-circuit conditions [1], [2], [3]. Though the finite difference methods seem to be the most adequate, they require, however, high computer elapse times and large memories, too. Consequently, the paper [4] suggests a decoupled method which, as it is stated, offers accurate enough prediction for the time-current characteristics, without excessive computation times and memories.
As it is customary, the current-limiting fuse-links include more fusible elements made up of multiple constrictions (reduced section) tapes. These constrictions can have various shapes: circular, rectangular,

trapezoidal.
The papers [5], [6], [7] present original methods for solving the general heating equation, in non-adiabatic regime, of the variable section fusible elements (tape-shaped of silver, with circular constrictions).
In the present paper, the general heating equation of the fusible elements is written for fusible elements made up of silver tapes with rectangular constrictions and an original method for solving this equation is also presented. The computation results are compared with the experimental ones obtained when testing some high-voltage current-limiting electric fuses, designed for power transformer (short-circuit and overload) protection.

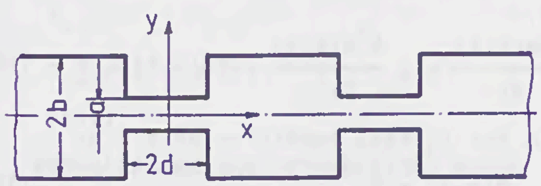

Fig. 1 Fusible element

2. Establishing the differential equation of a fusible element heating with rectangular constrictions

We consider a fusible element made up of a rectangular perforation tape, schematically presented in Figure 1.
If one takes into account both the heat conduction in the fusible element and the heat transfer by convection and if one assumes that the fusible element temperature varies with its length only (the element width is much smaller, while the thickness is negligible), then, one can obtain the following general equation for a variable section fusible element heating (in non-adiabatic regime):

$$\gamma c \frac{\partial \theta(x,t)}{\partial t} = \lambda \frac{\partial^2 \theta(x,t)}{\partial x^2} + \rho(\theta) . J^2(x,t) - \frac{l_x}{A_x} K . (\theta(x,t) - \theta_a) \qquad (1)$$

where:

$$\rho(\theta) = \rho_o(1 + \alpha_o \theta)$$

In the case of the fusible element in Fig.1 the current density $J(x,t)$ and the l_x/A_x ratio have the following expressions:

- for $o \leq x < d$:

$$J(x,t) = \frac{i(t)}{A_x} = \frac{i(t)}{2ag} \cong \frac{I_o}{2ag}$$

where:

$$A_x = 2ag; \quad l_x = 2(2a + g)$$

$$\frac{l_x}{A_x} = \frac{2(2a+g)}{2ag} \cong \frac{2}{g} \qquad (g \ll a)$$

- for $x \geq d$:

$$J(x,t) = \frac{i(t)}{A_x} = \frac{i(t)}{2bg} \cong \frac{I_o}{2bg}$$

where:

$$A_x = 2bg; \quad l_x = 2(2b + g)$$

$$\frac{l_x}{A_x} = \frac{2(2b+g)}{2bg} \cong \frac{2}{g} \qquad (g \ll b)$$

As one can notice, in the relations that determine the current density $J(x,t)$, the instantaneous value of the current $i(t) = \sqrt{2} I_o \sin\omega t$ was replaced by the overload current effective value, I_o, because the times are high enough for the thermal effect of the alternative current $i(t)$ to be the same as that of the steady current I_o.
Taking into account the above-given relations, the general equation (1) of a variable section fusible element heating, in the particular case of a fusible element made up of a rectangular constriction tape, will have the following two expressions:

- for $o \leq x < d$:

$$\gamma c \frac{\partial \theta(x,t)}{\partial t} = \lambda \frac{\partial^2 \theta(x,t)}{\partial x^2} + \rho_o(1+\alpha_o \theta(x,t)) . \frac{I_o^2}{4a^2 g^2} - \frac{2K}{g}(\theta(x,t) - \theta_a) \qquad (2)$$

- for $x \geq d$:

$$\gamma c \frac{\partial \theta(x,t)}{\partial t} = \lambda \frac{\partial^2 \theta(x,t)}{\partial x^2} + \rho_o(1+\alpha_o \theta(x,t)) . \frac{I_o^2}{4b^2 g^2} - \frac{2K}{g}(\theta(x,t) - \theta_a) \qquad (3)$$

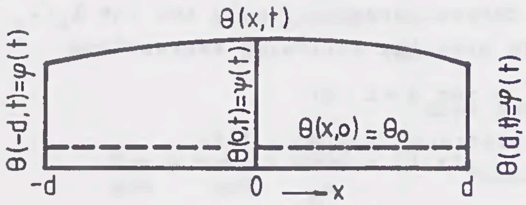

Fig. 2 Limiting conditions

3. Initial and limiting conditions

The problem finally raised is related to the determination of the time t_f necessary for the fusible element temperature $\theta(x,t)$ to reach its fusion temperature θ_f in the $x = o$ point, where the fusible element section is minimum, for various overload current values, I_o. To this purpose, one must solve the differential equation (2) - partial differential equation of a parabolic type - by means of certain initial and limiting conditions. They have the following form:

$$\theta(x,o) = \chi(x) \quad (4)$$

$$\theta(o,t) = \psi(t) \quad (5)$$

$$\theta(d,t) = \varphi(t) \quad (6)$$

and are schematically shown in Figure 2. These conditions were established in paper [8] and have the following expressions:

$$\chi(x) = \theta_o \quad (7)$$

$$\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=o} = o \quad (8)$$

$$\varphi(t) = \theta_f - (\theta_f - \theta_o)\exp(-t/T) \quad (9)$$

where $\theta_f = \theta_{max}$.

On the basis of the relations established for the boundary condition determination $\varphi(t)$ - established on the basis of the differential equation (3) - one has obtained the analytical expression of the heat transfer coefficient K by the lateral side, as well as of the time constant T:

$$K = \frac{\alpha_o \rho_o g}{2} \cdot \frac{I_o^2}{4b^2g^2} \cdot \frac{\theta_f - 1/\alpha_o}{\theta_f - \theta_a} \quad (10)$$

$$\frac{1}{T} = \frac{2}{\gamma c g} K - \frac{\alpha_o \rho_o}{\gamma c} \cdot \frac{I_o^2}{4b^2g^2} \quad (11)$$

One can notice that both the global coefficient K and the time constant T depend on the testing currents, as well as the fusible element sizes.

4. Algorithm for solving the fusible element heating equation

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If the differential equation (2) the following notations are used:

$$A = \frac{\lambda}{\gamma c} \quad (12)$$

$$B = \frac{1}{\gamma c}\left(\frac{\alpha_o \rho_o I_o^2}{4a^2g^2} - \frac{2}{g}K\right) \quad (13)$$

$$C = \frac{1}{\gamma c}\left(\frac{\rho_o I_o^2}{4a^2g^2} - \frac{2\theta_a}{g}K\right) \quad (14)$$

then, the fusible element heating equation can be written as:

$$\frac{\partial\theta(x,t)}{\partial t} = A\frac{\partial^2\theta(x,t)}{\partial x^2} + B\theta(x,t) + C \quad (15)$$

with the initial condition at $t = o$:

$$\theta(x,o) = \theta_o \quad (16)$$

and the limiting conditions:

$$\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=o} = o \quad (8)$$

$$\theta(d,t) = \theta(-d,t) = \varphi(t) \quad (17)$$

When writing the limiting condition (17), the symetry of the fusible element has been taken into account (see Fig. 2).
In order to solve the differential equation (15), a second order partial differential equation of a parabolic type, the DSCT numerical method (Discrete Space, Continuous Time) was used.
By discretizing the space x, the second order partial differential equation (15) is transformed into a first order ordinary differential equations system, a function of time t, which can be written in a concentrated form as:

$$\frac{d\theta_i(t)}{dt} = A\frac{\theta_{i+1}(t) - 2\theta_i(t) + \theta_{i-1}(t)}{\Delta x^2} + B\theta_i(t) + C \quad (18)$$

where $i = 1,\dots,n$; $\Delta x = d/n$ because, as it is shown in Figure 3, the space x discretization in the interval [o, d] was made by a constant step.

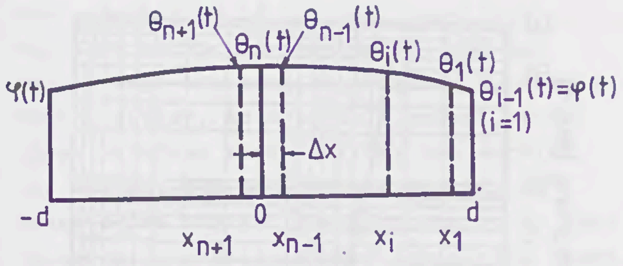

Fig. 3 Discretization of space x in the interval $[o, d]$.

The initial condition (16) is transformed in a set of initial conditions:

$$\theta_i(o) = \theta_o \qquad (19)$$
$$(i=1,\dots,n)$$

and the limiting conditions can be written as (see Fig. 3):

$$\theta_{n+1}(t) = \theta_{n-1}(t) \qquad (2o)$$

$$\theta_{i-1}(t) = \varphi(t) \qquad (21)$$
$$(i=1)$$

5. Establishing the mathematical model for the computer

In order to establish the mathematical model for the computer, the boundary function $\varphi(t)$ defined by the relation (9) was replaced by its derivative:

$$\frac{d\varphi(t)}{dt} = \frac{1}{T}(\theta_f - \varphi(t)) \qquad (22)$$

with the initial condition at $t = o$:

$$\varphi(t) = \theta_c \qquad (23)$$

On the basis of equations (18) and (22), in view of the relations (2o), (21) and assigning a certain value to the parameter n, one can obtain a system of N differential equations.

If the following notations are used:

$$y_1 = \varphi;\ y_2 = \theta_1;\ y_3 = \theta_2;\dots;\ y_N = \theta_n \qquad (24)$$

where $n = N-1$, the mathematical model for digital computer will have the following general form:

$$\frac{dy_1}{dt} = \frac{1}{T}(\theta_f - y_1)$$
$$\frac{dy_{j+1}}{dt} = A_o(y_{j+2} - 2y_{j+1} + y_j) + By_{j+1} + C \qquad (25)$$
$$\frac{dy_N}{dt} = 2A_o(y_{N-1} - y_N) + By_N + C$$

where: $j=1,\dots, N-2$

$$A_o = \frac{(N-1)^2}{d^2} A \qquad (26)$$

while the initial conditions are:

$$y_i(o) = \theta_o \qquad (27)$$
$$(i=1,\dots,N)$$

In order to solve the differential equation system (25) with the initial conditions (27), a computation program was carried out on a digital computer, using the integration method RUNGE-KUTTA-GILL.

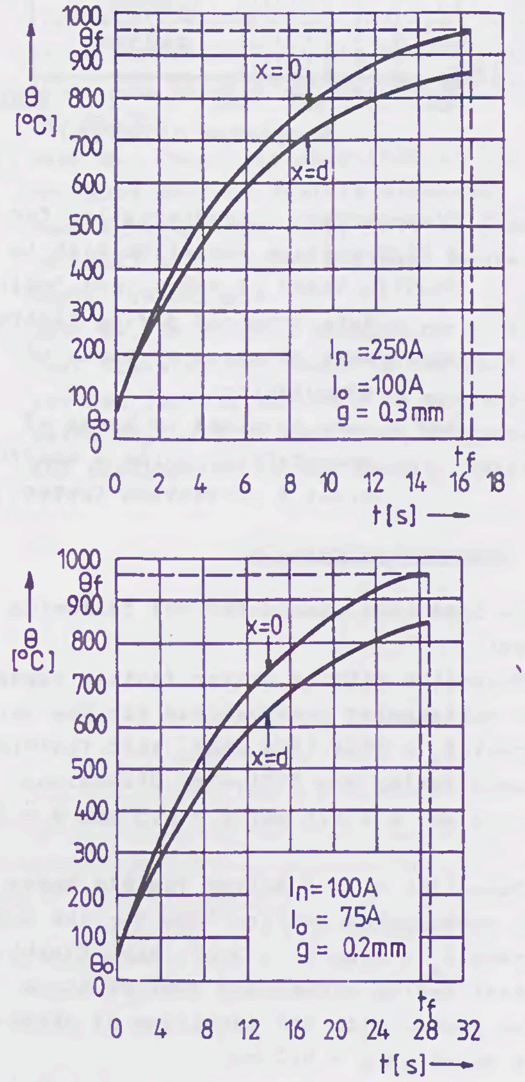

Fig. 4 Computation results.

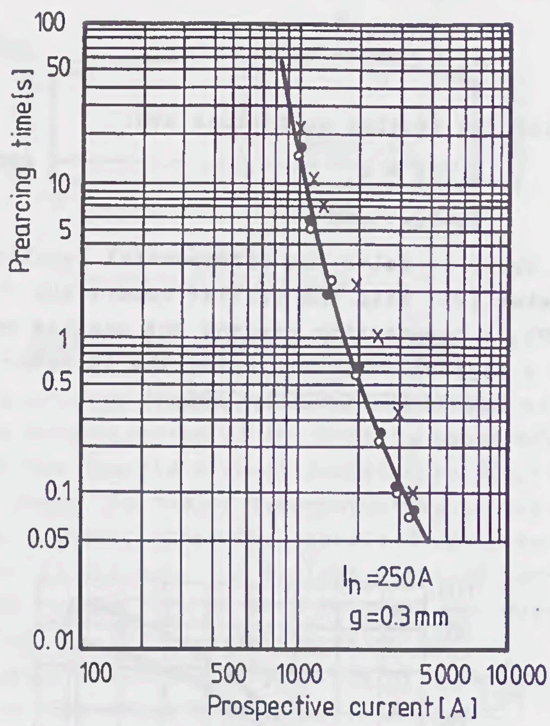

Fig. 5 Time-current characteristics for a high-voltage fuse-link with lo fusible tapes (I_n=25oA; g=o.3mm):
... - data obtained during testings
xxx - data obtained by means of computation
ooo - data obtained by means of computation, using a coefficient K correction factor.

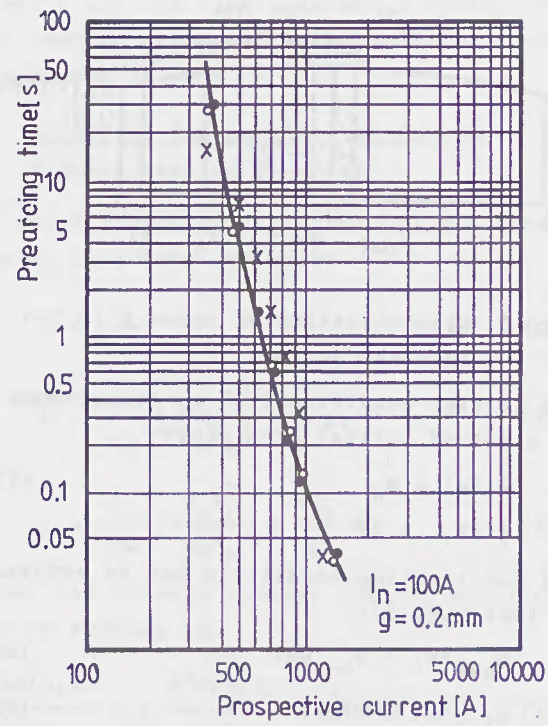

Fig. 6 Time-current characteristics for a high-voltage fuse-link with 5 fusible tapes (I_n=looA; g=o.2mm):
... - data obtained during testings
xxx - data obtained by means of computation
ooo - data obtained by means of computation, using a coefficient K correction factor.

6. Computation results

There have been considered the following cases:

a) Fuse-link with lo silver fusible tapes with rectangular perforations for the rated current I_n = 25oA (I = 25A), each fusible element having the following dimensions: g = o.3 mm; a = o.5 mm; b = 1.5 mm; d = 3 mm.

b) Fuse-link with 5 silver fusible tapes, with rectangular perforations for the rated current I_n = looA (I = 2oA), each fusible element having dimensions similar those given above, with the exception of thickness which is g = o.2 mm.

In each of the two cases, there were taken into account certain prospective currents, from the time-current characteristics, obtained during the fuse-link overload testing and there were computed the corresponding current by a single fusible element. For each of these oveload current values, I_o, there were established - by means of the digital computer - the temperature time variation characteristics $\theta = \theta(t)$ in various points of the interval [o, d], up to the moment t_f when the fusible element temperature in the x=o point reaches the silver fusion temperature, θ_f = 96o °C.

As an example, Figure 4 shows the time variation forms of temperature $\theta = \theta(t)$ for x = o and x = d, obtained in two computation variants. As one can notice, when analyzing the presented curves, in the constriction (reduced section) area of the fusible tapes, the temperature variations at the moment $t = t_f$ are of the order of tens of degrees ($\Delta\theta$ = 83°C and $\Delta\theta$ = 1o8°C, respectively).

Figures 5 and 6 present the time-current characteristics of the two fuse-links analyzed above. In both cases, one can notice that the results obtained by computation differ, to a certain extent, from those obtained when testings are carried out namely, for the same values of the prospective current one can obtain, by computation, generally higher pre-arcing times. However, it was found out that, if in the place of the heat transfer coefficient K, defined by the relation (1o), a corrected coefficient $K' = k_o K$ is used in the computations, where the correction factor k_o is defined by means of the empirical relation:

$$k_o = 1.1 + \frac{I_o - 1oo}{25} \cdot o.2 \qquad (28)$$

then, the obtained computation results are in full agreement with the experimental ones.

7. Conclusions

The heat transfer phenomena in current-limiting electric fuses are mathematically modelled by partial differential equations of a parabolic type. Due to the fact that there occur high prearcing times (of the order of seconds and tens of seconds) when heating, in an overload regime, the fuse-links designed for power transformer protection, the thermal phenomena cannot be considered as adiabatic.

The computation method presented in this paper makes it possible to establish the temperature curves $\theta = \theta(t)$ for x = parameter and, implicitly, to establish the time t_f when temperature $\theta(t)$ reaches the fusible element fusion temperature θ_f in the x = o point, where the fusible element section is minimum. The paper presents computation results obtained in various study variants which are compared with the experimental ones obtained during the testings (time-current characteristics).

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