APPLICATION OF THE ARC-PINCH-FORCES-INTERACTION THEORY TO THE CALCULATION OF STRIATION MODULUS OF THE STRIP H.B.C. FUSES

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1. INTRODUCTION

The striation is a term used to determine of the plain wire or plain strip disintegrating kind due to a h.b.c. fuse operation under short-circuit conditions. The origin of name "the striation" / introduced by Nasiłowski [1] / outcomes from the picture of the remainder after uniform cross-section fuse-element disruption, which a typical example is shown in Fig.1. The tubewise fulgurite after short-circuit current interruption does indicate a nonconductive hole streched between the fuse-link terminals. The hole is surrounded by the sintered quartz-sand grains mixed alternatively: with the fuse-element metal and then with the arc-plasma, according to the Fig.2. In result the photograph obtained after current interruption shows the typical streaks.

In our previous paper [3] the arc-pinch-forces-interaction / APFI / theory has been introduced to explain the number of arising streaks as result of wire disintegration. The paper also suggests the possible physics of the formation striated fulgurities.

It seems, the APFI-theory could also enlight the plain strip striation modulus according to the considerations given below.

2. CONDITIONS OF THE WHOLE STRIP STRIATION

The following Hibner´s experimental conditions [4] shall be fulfilled in order to transmit the whole fuse-element strip into the streaks:

-the current-density j_t in kA/mm^2 by the instant of streak disintegration shall respond to the relation

$$j_t \geqslant \frac{4.6}{S^{0.29}} \qquad /1/$$

where S- the strip cross-section in mm^2;

-the circuit specific energy, given by the ratio of the magnetic energy stored in circuit in the disintegrating instant t_t to the strip volume, shall satisfy the following inequality

$$\mathcal{E} \geqslant 25 \frac{W}{mm^3} \qquad /2/$$

If one of those conditions is not preserved, a part or several parts of the strip only passes into the streaks.

Usually the second condition is easy to satisfy, for only a fraction of the magnetic energy stored in circuit is sufficient to fit the inequality /2/. This is better described in the paper [3] for a wire striation, but for a strip striation the quantitive conditions are analogous. The more critical is

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the condition defined by the relation /1/. In here rather the large prospective short-circuit currents are necessary, which in rare practical cases have take place, specifically for h.b.c. fuses above say 300A rated current.

3. SUMMARY OF APFI-THEORY

A brief summary of the APFI-theory given in [3] is shown in the Fig.3, but now reffering to the strip striation. The essence of that theory is following: the number of pieces arising due to the strip disintegration is dictated by the equilibrium of the forces

$$F_a = F_{px} + F_{py} \qquad /3/$$

in which F_a - the arc-force in a disruption directed along the strip-axis, F_{px} - the pinch-force acting perpendicularly to the strip-piece adges, F_{py} -the pinch-force presses also perpendicularly but along the flat surfaces of that piece. All these forces are working in pairs in opposite directions compressing the strip-piece. Obviously, the strip-piece by the instant t_t is completely liquified and is not so regular as it outcomes from the Fig.3. The flat surfaces in reality are wrinkled due to neighbouring sand grains. The slots in which the arcs are acting are not so regularly directed perpendicular to the strip-axis. But for the clearness we cosider a regular model agreed with shown in the Fig.3. Indicated in it imagination reffers to an instant existing just before the streak formation. Here all strip-pieces and all arcs are still in place of the initial strip.

4. SIMPLIFIED EVALUATION OF THE STREAKS MODULUS

A typical time duration of the transition of a strip into the streaks at normal short-circuit conditions of an a.c. network is of the several decades or hundreds microseconds. It is far shorter than the short-circuit time-constant. Hence one can assume the current magnitude / called in further "the transition current"/ constant throughout the whole period of striation. The value of the transition current i_t , which is nearly equal to the cut-off current of a common h.b.c. fuse, is to determine using the following relations [5,6] :
for a.c.

$$i_t \cong 11 \sqrt[3]{S^2 K \cdot I_p} \qquad /4/$$

and for d.c.

$$i_t \cong 1{,}4 \sqrt[3]{\frac{S^2 K \cdot I_u}{T}} \qquad /5/$$

in which K- the material Mayer's constant, I_p- the prospective current, I_u- the steady-state short-circuit current, T- the time-constant.

Assuming the constant strip-thickness h and denoting the strip width by a one can easy rearrange above given relations as follows

$$i_t \cong C_1 \cdot a^{2/3} \cdot I_p^{1/3} \quad \text{for a.c.} \qquad /6/$$

$$i_t = C_2 \cdot a^{2/3} \cdot I_u^{1/3} \quad \text{for d.c.} \qquad /7/$$

in which C_1 and C_2 are some constants.

The arc-pressure is proportional to the current density square, assuming simplification that the arc cross-section is equal to the strip cross-section

$$p_a = k \cdot j_t^2 \qquad /8/$$

where j_t - the transition current density, k- some constant. Remembering that $i_t = j_t \cdot S$ from /6/, /7/ and /8/ follows

$$j_t^2 = A \cdot a^{-2/3} \qquad /9/$$

and

$$p_a = B \cdot a^{-2/3} \qquad /10/$$

Thus the force F_a depends on the width at constant strip´s thickness in a very simple form, viz.

$$F_a = p_a \cdot a = B \cdot a^{1/3} \qquad /11/$$

On the other hand the forces F_{px} and F_{py} per piece-length of the strip are defined by the relations derived in Appendix, which are rewritten below replacing i by i_t

$$F_{px} = \frac{\mu_0 i_t^2 \lambda}{4\pi a}\left(\ln \frac{4a^2 + h^2}{a^2 + h^2} + 2\frac{a}{h}\operatorname{arctg}\frac{h}{2a} - \frac{a}{h}\operatorname{arctg}\frac{h}{a}\right) \qquad /12/$$

$$F_{py} = \frac{\mu_0 i_t^2 \lambda}{4\pi h}\left(\ln \frac{4h^2 + a^2}{h^2 + a^2} + 2\frac{h}{a}\operatorname{arctg}\frac{a}{2h} - \frac{h}{a}\operatorname{arctg}\frac{a}{h}\right) \qquad /13/$$

where λ is the strip-piece length, which approximatively is equal to the modulus of striation, because the initial length of the two-sided arcs is practically negligible in comparison with the length of the strip-piece itself.

Calculations for the practical range of ratios a/h, given in the chapter 5 in form of the examples, does indicate that the both forces are approximatively equal, hence it follows

$$F_{px} + F_{py} \cong 2F_{px} \qquad /14/$$

For a thin strip the relation /12/ gets the form

$$F_{px} = \frac{\mu_0 i_t^2 \lambda}{2\pi a} \ln 2 \qquad /15/$$

Connecting dependances /6/, /7/ and /15/ easy is to write for both a.c. and d.c.

$$F_{px} = N' a^{1/3} \lambda \qquad /16/$$

hence

$$2F_{px} = N a^{1/3} \lambda \qquad /17/$$

where N' and N - the constants.

From relations /2/, /11/, /14/ and /17/ yields the following striation modulus

$$\lambda = \frac{M}{N} = \text{const} \qquad /18/$$

If the strip thickness is kept constant then the S-changes are proportional to the changes of the strip width a. So arise the following conclusion: the striation modulus of the thin strips is independent from the cross-sectional area.

While the Hibner´s experiments [4] made on strips of thickness 0.05-0.2mm and widths 2.5-15mm embedded in quartz sand of granularity 0.3-0.5mm and of standart packing density indicates on the following average modulus

$$\lambda_e = 3{,}1 \cdot S^{0,3} \qquad /19/$$

Hence the exemplary errors outcoming from the relation /18/ in juxtaposition to the /19/ are as follows: for the 2 times greater strip-width the experimental modulus is abt 23% larger than according to APFI-theory, for the 3 times- it is abt 40% larger, but for 5 times,- already abt 63% larger.

Nevertheless the indicated errors, specifically for the practical range of the a/h ratio variations by this same S, are not so large ones taking into considerations the very simple model considered in this chapter.

5. INFLUENCE OF RATIO a/h ON THE STRIATION MODULUS

The formula /19/ does suggest the independence of the modulus on the ratio a/h by this same S. So arise an essential question, why only the cross-section but not the a/h-ratio has the influence on that modulus.

Looking on the possible answers one would suggest as an reason of the mentioned independence a game of the arc pressure relief and the pinch-forces for different a/h ratios.

Reffering to the arc-pressure relief logically is that larger the a/h ratio greater the arc-pressure relief. It outcomes from the greater circumferences of the strip at larger ratios a/h by this same strip´s cross-sectional area. Greater the strip´s circumference larger the resultant cross-sectional area S_i of the interstices between the sand grains surrounded the strip.

Johann´s qualitative relation on the pile up pressure of the arc channel existing during the arcing in a h.b.c. fuse [7] does´nt fit the situation in here under discussion, because we have to consider the very initial arcing stage. The plasma outflow into the interstices has been just started only, whereas in Johann´s model a part of the arc-plasma is already in the sand of the thickness equalling to the fulgurite thickness. Hence it seems more correct is to assume that the pressure relief degree is invers proportional to the S_i. This assumption accords with the proportionality of the pressure relief to the gas-dynamic time-constant of the considered plasma outflow into the sand. This time-constant by this same strip´s cross-section is reciprocal to the S_i, assuming for different a/h ratios this same initial outflow velocity. Because the current i_t for different a/h is also practically exactly this same it is easy to conclude, for the adiabatic heating, that there are the same outflow starting conditions irrespectively of the area S. In result during this same very short time period, over which the relation /3/ leads to the defined number of streaks, the following proportionality shall be valid

$$p_a \sim \frac{1}{S_i} \sim \frac{1}{c} \qquad /20/$$

where c is the strip's circumference. But for thin strips one can write

$$p_a \sim \frac{1}{c} \sim \frac{1}{a} \qquad /21/$$

On the other hand the pinch-forces for the thin strips done by the relation /15/ are also reciprocal to the dimensions a. That's why, if for a certain dimension a this relation has been fulfilled, then for any arbitrary a it is also valid. But the obvious condition is that the a/h is rather large one, say several decades. Hence one can conclude that for thin strips the striation modulus should be independent of their cross-sectional area. In turn, if the strip thickness h is not negligible in comparison with its width a, the relations /12/ and /13/ shall be applied. It comes out even here that the pinch-forces again do fit the inverse proportional relation to the dimension a, however with the some approximation. The following examples could demonstrate this approximation.

Example 1. $S = 0.625 mm^2$ = const but we have two different strips: 1/$a \times h = 2.5 mm \times 0.25 mm = 0.625 mm^2$, $a/h = 10$ and 2/$a \times h = 5 mm \times 0.125 mm = 0.625 mm^2$, $a/h = 40$.

Results of calculations in the case 1/ are: from equation /12/ $F_{px1} = 0.55 B_1$ and from equation /13/ $F_{py1} = 0.52 B_1$, where $B_1 = \frac{\mu_0 i^2}{4\pi}$

Results of calculations in the case 2/ are: $F_{px2} = 0.273 B_1$, $F_{py2} = 0.313 B_1$.

The circumference ratio $c_1/c_2 = 1.37$. But the ratios $F_{px1}/F_{px2} = 2$ and $F_{py1}/F_{py2} = 1.67$. So the conclusion is that the diminishing of the pinch-forces in the case 2/ in comparison with the case 1/ is approximatively compensated by the force F_a lowering. The latter lowering is proportional to the circumference ratio.

Example 2. Case 1/: $a \times h = 3.75 mm \times 0.2 mm = 0.75 mm^2$, $a/h = 18.8$. Case 2/: $a \times h = 15 mm \times 0.05 mm = 0.75 mm^2$, $a/h = 300$. Results of calculations in case 1/ are: $F_{px1} = 0.367 B_1$, $F_{py1} = 0.335 B_1$. Results of calculations in case 2/ are: $F_{px2} = 0.0925 B_1$, $F_{py2} = 0.105 B_1$. Then the ratio $c_1/c_2 = 3.9$. But the ratios $F_{px1}/F_{px2} = 3.98$ and $F_{py1}/F_{py2} = 3.2$. Again the pinch-forces diminishing is nearly compensated by the force F_a lowering.

6. FINAL REMARKS

Despite some differences between the results described by the theoretical relation /18/ and experimental one /19/ it seems that herein given speculations one can recognize as a possible explanation of the magnitude of the modulus of the strip striation occuring under short-circuit current in the h.b.c. fuses The physics of the formation of the rythmical fulgurite structure shown in the Fig.2 is rather similar to that of the wire striation [3] , thus it was'nt described here again.

7. ACKNOWLEDGMENT

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8. REFERENCES

[1] Nasiłowski J.: Unduloids and striated disintegration of wires. Exploding Wires. N.Y. 1964, vol.3. Plenum Press. Editors Chase W.G., Moore H.K.

[2] Hibner J.: Gradient resistance method of calculations of the overvoltage crest value generated by the short-circuit current-limiting fuses. / in Polish /. Ph.D.Thesis. Gdańsk Technical University. 1973.

[3] Lipski T.: On the theory of the striated fuse-wire disintegration. IEEE Trans. vol.PS-10. No4. Dec. 1982. p.339.

[4] Hibner J.: The number of arcs initiated during striated disintegration of the uniform strip h.b.c. fuse-elements at a.c. short-circuit current interruption. Int.Conf. on " The Electric Fuses and Their Applications". Trondheim. 1984.

[5] Hibner J.: Calculative method of determination of the cut-off current characteristics of fuses at a.c. / in Polish/. Wiad.Elektr. 1980. No.1, p.7.

[6] Hibner J.: Determination of the cut-off current characteristics of fuses by calculative method. P.I.D.c. current. Wiad.Elektr. 1978.No.3. / in Polish/

[7] Johann H.: Interrupting arcs in electric fuses with grained quenching medium./ in German /. Siemens Forsch.- u. Entwickl.- Ber. vol.10.Part 3. 1981. No.3. p.139.

Fig.1 Photograph of fulgurite after short-circuit current interruption by a h.b.c. fuse within strip element axh = 5mmx0.1mm. Circuit conditions: I_p = 3.8kA /RMS/, U = 230V, 50Hz, p.f. = = 0.5 [2]

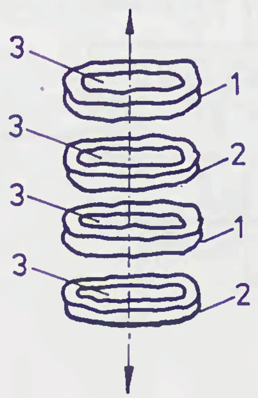

Fig.2 Situation after strip disintegration. Arrows show the original wire axis /1/ parts filled up by the arc plasma mixed with sand, /2/ parts filled up by the molten metal mixed with sand and /3/ nonconcuctive holes

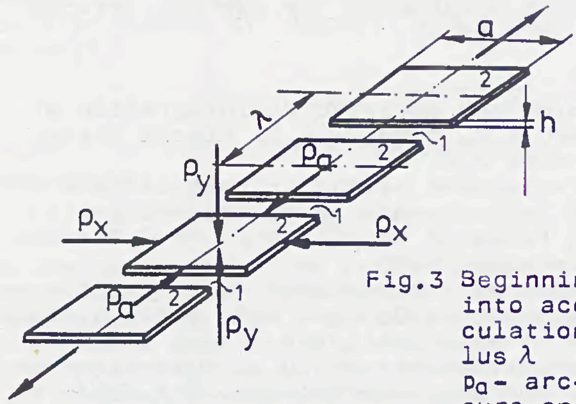

Fig.3 Beginning of the striation taken into account in simplified calculations of the striation modulus λ
p_a - arc-pressure; p_x - pinch-pressure on edges; p_y - pinch-pressure on flats; 1- slots with arcs; 2-strip pieces; a,h,- dimensions

APPENDIX

We consider pinch-forces acting on the edges F_{px} and the flats F_{py} of a fuse-link strip according to the Fig.1A. The flux density differential in both axis in the point / x_o, y_o / can be described as follows

$$dB = \frac{\mu_o I\, d_x\, d_y}{2\pi a h\left[(x_o-x)^2+(y_o-y)^2\right]^{1/2}}$$

$$dB_x = dB\,\frac{x_o-x}{\left[(x_o-x)^2+(y_o-y)^2\right]^{1/2}}$$

$$dB_y = dB\,\frac{y_o-y}{\left[(x_o-x)^2+(y_o-y)^2\right]^{1/2}}$$

Assuming in approximation

$$B_x(x_o,y_o) \cong B_x(0,y_o)$$

$$B_y(x_o,y_o) \cong B_y(x_o,0)$$

one can get the following relations for the flux densities

$$B_x(x_o,y_o) = \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{h}{2}}^{\frac{h}{2}} dB_x = \frac{\mu_o I}{4\pi a h}\left\{(x_o+\frac{a}{2})\ln\frac{(x_o+\frac{a}{2})^2+(y_o+\frac{h}{2})^2}{(x_o+\frac{a}{2})^2+(y_o-\frac{h}{2})^2}\right. +$$

$$+(x_o-\frac{a}{2})\ln\frac{(x_o-\frac{a}{2})^2+(y_o-\frac{h}{2})^2}{(x_o-\frac{a}{2})^2+(y_o+\frac{h}{2})^2} + (2y_o+h)\,\mathrm{arctg}\,\frac{a(y_o+\frac{h}{2})}{(y_o+\frac{h}{2})^2+(x_o^2-\frac{a^2}{4})} -$$

$$\left. -(2y_o-h)\,\mathrm{arctg}\,\frac{a(y_o-\frac{h}{2})}{(y_o-\frac{h}{2})^2+(x_o^2-\frac{a^2}{4})}\right\}$$

$$B_y(x_0 y_0) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} dB_y = \frac{\mu_0 I}{4\pi a h} \Bigg\{ (y_0 + \frac{h}{2}) \ln \frac{(x_0+\frac{a}{2})^2 + (y_0+\frac{h}{2})^2}{(x_0-\frac{a}{2})^2+(y_0-\frac{h}{2})^2} +$$

$$+ (y_0 - \frac{h}{2}) \ln \frac{(x_0-\frac{a}{2})^2+(y_0-\frac{h}{2})^2}{(x_0+\frac{a}{2})^2+(y_0-\frac{h}{2})^2} + (2x_0+a)\operatorname{arctg} \frac{h(x_0+\frac{a}{2})}{(x_0+\frac{a}{2})^2+(y_0^2-\frac{h^2}{4})} -$$

$$- (2x_0-a)\operatorname{arctg} \frac{h(x_0-\frac{a}{2})}{(x_0-\frac{a}{2})^2+(y_0^2-\frac{h^2}{4})} \Bigg\}$$

It can be demonstrated that the errors by that assumptions is not greater than 10%.

The total average pinch-forces are described by the following dependances

$$F_y = l \int_0^{\frac{h}{2}} B_x \, dI'$$

$$dI' = \frac{I}{\cancel{\phi} h} \cancel{\phi} \cdot dy_0 = \frac{I}{h} dy_0$$

$$F_y = \frac{Il}{h} \int_0^{\frac{h}{2}} B_x(y_0) dy_0 = \frac{\mu_0 I^2 l}{4\pi h} \left[\ln \frac{4h^2+a^2}{h^2+a^2} + 2\frac{h}{a} \operatorname{arctg} \frac{a}{2h} - \frac{h}{a} \operatorname{arctg} \frac{a}{h} \right]$$

$$F_x = l \int_0^{a/2} B_x \, dI''$$

$$dI'' = \frac{I}{a \cancel{h}} \cancel{h} \cdot dx_0 = \frac{I}{a} dx_0$$

$$F_x = \frac{Il}{a} \int_0^{a/2} B_x \, dx_0 = \frac{\mu_0 I^2 l}{4\pi a} \left[\ln \frac{4a^2+h^2}{a^2+h^2} + 2\frac{a}{h} \operatorname{arctg} \frac{h}{2a} - \frac{a}{h} \operatorname{arctg} \frac{h}{a} \right]$$

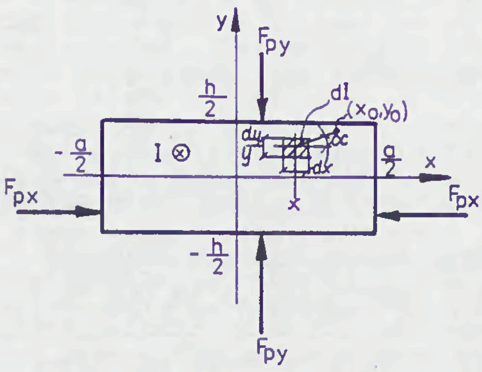

Fig.1A