Usage of I^2t and reference formulae for key waveshapes

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***Abstract*—The use of the I^2t integral in fuse design and application is well established, but care is needed in its use. In this paper the limitation of the adiabatic assumption for the prearcing I^2t is illustrated, and the advantages using the virtual r.m.s. current I_V are discussed. Formulae are given for waveshapes found in practical applications, which should provided a useful resource for fuse application engineers.**

***Keywords*--Fuse prearcing I^2t. Virtual r.m.s. current. I^2t for key waveshapes.**

I. Introduction

In practical applications the fuse I^2t integral can be used in different ways. In one example a fuse may be required to withstand a surge without operating. In another case the total let-through I^2t of the fuse under fault conditions may need to be low enough to provide protection to an associated device or system. Although these concepts are well established, care is needed in their application. The use of the virtual r.m.s current I_V for fuse characteristics and in applications is discussed.

Formulae for the I^2t integral are given for wave shapes found in practical applications. They are divided into two groups:

- simple waves of finite duration for which the I^2t value is constant
- waves of infinite duration for which the I^2t value is given as a function of time, including AC short-circuit faults, DC short-circuit fault, 3-phase AC capacitor switching-in and capacitor bank discharge

Typical conditions of usage are discussed. Many of these formulae are well known, but some do not appear to have been published before. The collection of them all together in this paper should provide a useful resource for fuse application engineers.

II. Adiabatic melting I^2T of fuse elements

If the heating of a fuse element with cross-section S is very rapid, such that heat losses from the element can be neglected, all of the heat generated due to the current flow is used to raise the temperature of the element, and the melting time t_m can be calculated from

$$\int_0^{t_m} i^2\,dt = K_M S^2 \tag{1}$$

where K_M is known as **Meyer's constant** [1]. For a specified ambient temperature K_M depends only upon the thermophysical properties of the element. K_M is given by $(mc/\alpha\rho_0)\ln(1+\alpha\theta_m)$ where for the element material:

m = specific gravity, gm-mm^{-3}
c = specific heat, J-gm^{-1}-degC^{-1}
ρ_0 = specific resistivity at ambient temperature, Ω-mm,
α = temperature coefficient of resistivity, degC^{-1}
θ_m = temperature rise required to reach melting

Table I given typical values of the thermophysical properties of metals and alloys commonly used for fuse elements usedin power fuses and miniature fuses and the calculated value of K_M.

TABLE I. Properties of fuse element materials

Material	ρ_{20} Ω-mm x 10^{-5}	α degC^{-1} x 10^{-3}	c J/g/degC	m g/mm^3 x 10^{-3}	T_M degC	K_M A^{2}s/mm^4
Silver	1.62	4.45	0.255	10.49	960.5	61069
Copper	1.71	3.95	0.438	8.92	1083	95187
Zinc	6.00	4.90	0.419	7.10	420	10973
Brass	6.16	1.60	0.377	8.53	955	29844
Nickel silver	31.0	0.257	0.378	8.70	1035	9566
Nichrome	112.2	0.097	0.435	8.34	1350	4044
Phosphor bronze	15.7	0.60	0.377	8.80	1083	17375

The values given for alloys in Table I should be regarded as approximate, as they depend on the exact composition. Fuse designers commonly use K_M values of 80,000 for silver and 100,000 for copper, which are somewhat higher than given in Table I. This is partly because element disintegration requires a little more energy to be supplied after melting point has been reached, but more important is the fact that truly adiabatic melting only occurs for very short melting times.

Fig. 1 shows the ratio of actual prearcing I^2t to the adiabatic value for a plain copper strip surrounded by sand in a UL standard 200A fuse tube. This curve was calculated using the methods described in [2] with 50Hz symmetrical sine wave as the test current. For high test currents (short prearcing times) the heating is adiabatic, but for longer times the ratio starts to rise, due to heat loss from the element to the surrounding sand. For a time t^* of about 150ms the prearcing I^2t is about 20% higher than the adiabatic value.

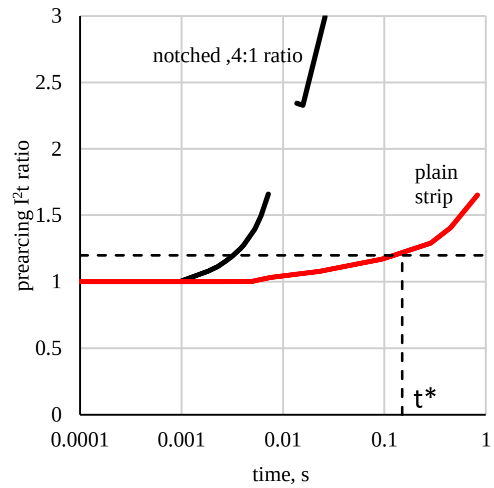


Fig.1 Dependence of prearcing I^2t on time

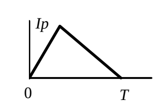
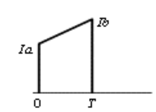
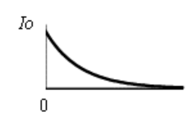
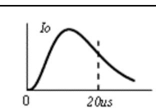
When a semicircular notch with a 4:1 ratio of full width to reduced width is introduced, the value of t^* falls to about 3ms. This is caused by transient conduction heat loss from the notch to the surrounding full element cross-section, and true adiabatic heating only occurs for times less than 1ms. A ratio of 4:1 is typical for a fuse for general power applications. Fuses for the protection of semiconductors use higher ratios, and adiabatic conditions can only be assumed for times much lower than 1ms.

The prearcing I^2t for the notched element rises rapidly for times greater than 3ms, and there is a large gap in the curve between about 7ms to 12 ms. As the test current approaches the first current zero at 10ms, the notch temperature falls rapidly, making it impossible for the notch to melt near to the current zero. A similar gap is present in the curve for the plain strip, because of heat loss to the sand. However this is so small that the gap is not visible on Fig.1. In practice an a.c fault (or test) current, may not be symmetric, and the gaps can be found anywhere surrounding a current zero.

III. Simple Pulses with Finite I^2T

Table II gives the I^2t values for some common cases. These are often used in applications where the fuse must withstand a short-duration surge without damage, and typically the I^2t of the surge should be less than about 60% of the prearcing I^2t of the fuse. However for this method the surge duration T must be less than t^*.

TABLE II. I^2T FOR SIMPLE PULSES

Description	Shape	I^2t
Triangular		$\frac{1}{3}I_p^2T$
Trapezoidal		$\frac{(I_a^2 + I_aI_b + I_b^2)\,T}{3}$
Exponential decay $i = I_0 e^{-t/\tau}$		$\frac{1}{2}I_0^2\tau$
8/20 µs surge $i = I_0 A\, t^3 e^{-t/\tau}$		$\frac{720\, I_0^2 A^2 \tau^7}{128}$

The trapezoidal case is included here because it can be used a building block to construct complex nonstandard waveshapes.

The 8/20µs shown here is the IEEE/ANSI model [3]. This is a mathematical formula which gives a unidirectional pulse which meets the requirements of a standard wave if A=0.01243 and τ = 3.911µs. However practical test circuits using a capacitor bank discharge always give an oscillatory wave, and the formula given in section VII should be used for this case.

IV. Use of the Time-Current Characteristic

Using the time-current characteristic for fuse applications is better than using curves like those shown in Fig.1. However, there is still widespread misunderstanding of how these characteristics should be used. Historically it was believed that prearcing time should be plotted on the vertical axis, and that the prospective (available) current should be plotted on the horizontal axis (since this value is set by the test circuit and is a truly independent variable).

However this approach does not work for current-limiting fuses for short melting times. Fig. 2 shows curves plotted for a typical 200A industrial fuse. Two curves are shown. The first is for a symmetrical sine-wave, and gaps can be seen as explained in section II. The second is for a closing angle which gives a fully asymmetrical wave, with a circuit p.f. of 0.1. If the fuse just fails to melt within the first loop of current there is a long delay of many cycles until it does. There is a large spread between the two curves, and when all possible circuit closing angles are considered the characteristic becomes a widespread zone up to about 0.2 s.

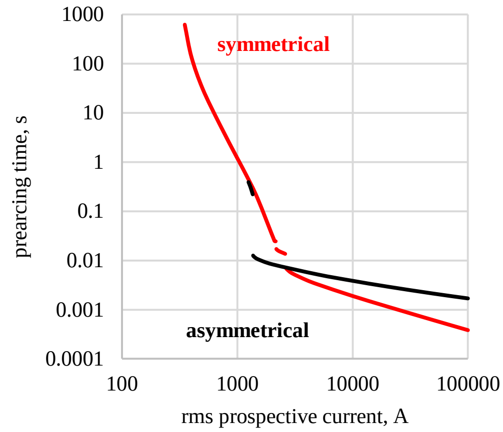


Fig. 2 time versus prospective (available) current

When this fuse is tested at the maximum breaking current of 200kA the notches melt after about 0.25ms and the peak current is limited to about 22kA. The waveshape is roughly triangular and the r.m.s. value of the **current which actually flows through the element and causes it to melt** is 22/√3 =13kA. This is the point (13kA, 0.25ms) which should be plotted on the time-current characteristic, not (200kA, 0.25ms). Some fuse standards deal with this problem by ignoring it, but nowadays most acknowledge that the current plotted on the x-axis should be the "r.m.s.prearcing current". In the past this has also been given other names, but here it will be referred to as **virtual current**, defined by:

$$I_V(t) = \sqrt{\frac{\int_0^t i^2 dt}{t}} \qquad (2)$$

Fig. 3 shows the same data but with virtual current plotted on the x-axis. The spread has not been completely eliminated, but greatly reduced, and this allows a time-current characteristic to be generated which is useful for any type of fault current waveshape.

However when coordinating the characteristics of fuses and other protective devices the situation is often confused. Examples like Fig.4 can be found, which illustrates the protection of a device by an overload relay with a circuit breaker for switching and a fuse to give short-circuit protection. Also shown is a surge which the protective devices are supposed to withstand. Fig.4 is useful for illustrative purposes, but the x-axis is simply marked "current" and the curves plotted will not necessarily coordinate properly because definitions have been implicitly assumed.

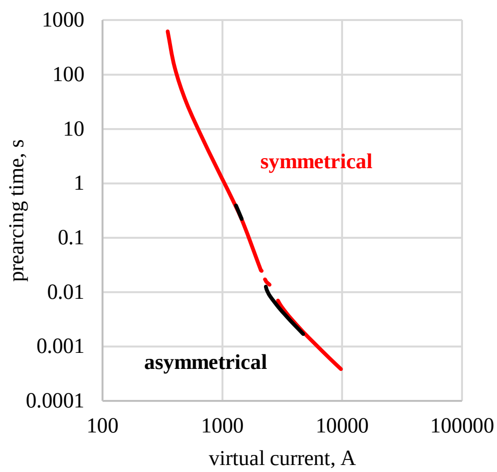


Fig.3 time versus virtual current

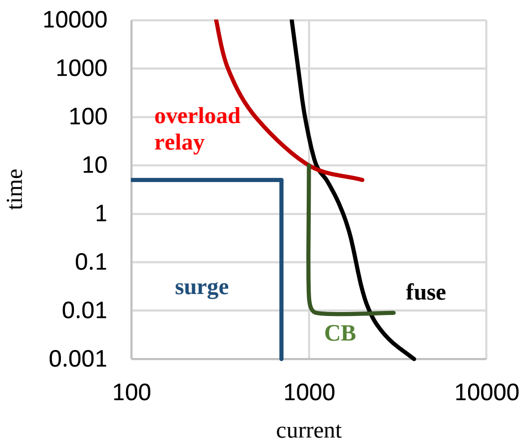


Fig.4 Typical protection example

The fuse characteristic shown uses virtual current on the x-axis, and the overload relay will coordinate correctly as it is a thermal device and for times greater than 1s the virtual current is identical with the r.m.s. prospective current.

The surge shown is supposed to represent a constant current of 700A which drops to zero after 5 seconds. However, what has been plotted is the **instantaneous** current as a function of time, rather than the virtual current. After the surge current falls to zero, the heating effect of the surge continues. For a surge of duration T with a fixed I^2t value C the virtual current for $t > T$ is $\sqrt{C/t}$ using (2) so I_V decreases as 1/√t.

Circuit breaker time-current characteristics use r.m.s. prospective (available) current on the x-axis which is different again and results in a large spread in operating times for high currents (not shown). So Fig. 4 mixes three different definitions of current (virtual, prospective current and instantaneous) on the same chart.

Fig.5 shows how a realistic representation of the heating effect of an AC surge can be plotted using virtual current. The case shown is an initial surge of 60A which falls to a constant 10A after 5s. The frequency is 50Hz and the circuit is closed at the instant which gives maximum offset in the transient current. Note that it takes more than 1000s for the virtual current to fall to 10A.

Virtual currents can be calculated for many common waveshapes using the formulae given in section VII of this paper. The first step is to calculate the I^2t integral at a time t using the formulae given, and then $I_V(t)$ is calculated using equation (2).

V. Conclusions

The melting of current-limiting fuses with notched elements is adiabatic only for very short melting times, well below 1ms. The prearcing I^2t increases with time and is greatly affected by the waveshape of the current. These effects are minimised by fuse manufacturers by using the virtual current to generate published time-current characteristics. However, proper coordination with surges and other devices requires consistency. The data given here in section VII should provide a useful resource for fuse application engineers. Most of these formula are well known but others do not appear to have been published before.

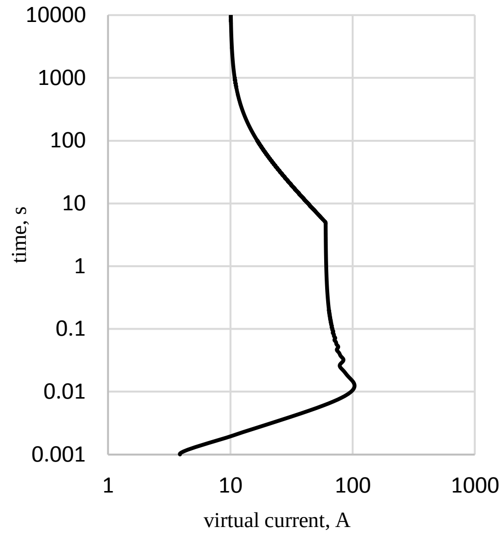


Fig.5 Virtual current for an a.c. surge

VI. References

[1] R. Rüdenberg. *Transient Performance of Electric Power Systems.* (M.I.T. Press, 1970).

[2] R.B. Standler. *Protection of electronic circuits from overvoltages*. (John Wiley, 1989)

[3] R.Wilkins, D. Suuronenen and L. O'Shields, "Developments in the Modelling of Fuse Breaking Tests", *4th ICEFA, Nottingham, UK, 1991*, pp 211-215.

VII. Appendix : I^2T for common waveshapes

Type	$\boldsymbol{i(t)}$	$\int_0^t i^2 dt$
DC short-circuit I = prospective current τ = source time constant	$i = I(1 - e^{-t/\tau})$	$I^2\left(t - \frac{3}{2}\tau - \frac{\tau}{2}e^{-2t/\tau} + 2\tau e^{-t/\tau}\right)$ $for\ \ t/\tau < 0.01\ use\ I^2\left[\frac{1}{3}\left(\frac{t}{\tau}\right)^3 - \frac{1}{4}\left(\frac{t}{\tau}\right)^4\right]$ $see\ Note\ 2$
AC short-circuit I = r.m.s. prospective current, ω = angular frequency φ = source power factor angle, θ = closing angle	$i = \sqrt{2}I\,\{\sin(\omega t + \theta - \varphi) - \sin(\theta - \varphi)e^{-\omega t/tan\varphi}\}$	$I^2\left[t - \frac{1}{2\omega}\{\sin 2(\omega t + \theta - \varphi) - \sin 2(\theta - \varphi)\} + \frac{tan\varphi}{\omega}sin^2(\vartheta - \varphi)(1 - e^{-2\omega t/tan\varphi}) + \frac{4sin\varphi}{\omega}\sin(\theta - \varphi)\{e^{-\omega t/tan\varphi}\sin(\omega t + \theta) - sin\theta\}\right]$ $for\ \omega t < 2\pi * \left(\frac{5}{360}\right)\ use\ \ 2I^2\omega^2\left[\frac{a^2t^3}{3} - \frac{2abt^4}{4} + \frac{b^2t^5}{5}\right]$ $see\ Note\ 2$ $where\ \ a = \cos(\theta - \varphi) + \sin(\vartheta - \varphi)/\tan\varphi \quad b = \omega\sin(\theta - \varphi)\left[\frac{1}{2} - \frac{1}{tan^2\varphi}\right]$

Type	$i(t)$	$\int_0^t i^2 dt$
Capacitor bank discharge C = capacitance, L = series inductance, R = resistance V = initial capacitor voltage $\alpha = R/2L$, $\omega_0^2 = 1/(LC)$	**1. oscillatory** $\alpha^2 < \omega_0^2$ $\omega_n = \sqrt{\omega_0^2 - \alpha^2}$ $i = \frac{V}{\omega_n L} e^{-\alpha t} \sin \omega_n t$ **2. critically damped** $\alpha^2 = \omega_0^2$ $i = \frac{V}{2L} t e^{-\alpha t}$ **3. overdamped** $\alpha^2 > \omega_0^2$ $a = \propto - \sqrt{\alpha^2 - \omega_0^2}$ $b = \propto + \sqrt{\alpha^2 - \omega_0^2}$ $i = \frac{V}{2L\sqrt{\alpha^2 - \omega_0^2}} [e^{-at} - e^{-bt}]$	$\frac{1}{2}\frac{CV^2}{R} - \frac{1}{4}\left(\frac{V}{\omega_n L}\right)^2 e^{-2\alpha t}\left[\frac{1}{\alpha} - \frac{\alpha}{\omega_0^2}\cos 2\omega_n t + \frac{\omega_n}{\omega_0^2}\sin 2\omega_n t\right]$ for short times ($\omega_n t < 2\pi*(5/360)$ use $\frac{1}{3}\left(\frac{V}{L}\right)^2 t^3$ *see Note* 2 $\frac{1}{2}\frac{CV^2}{R} - e^{-2\alpha t}\left[\frac{1}{4\alpha^3} + \frac{t}{2\alpha^2} + \frac{t^2}{2\alpha}\right]$ $\left(\frac{V}{2L\sqrt{\alpha^2 - \omega_0^2}}\right)^2 \left[\frac{1 - e^{-2at}}{2a} + \frac{1 - e^{-2bt}}{2b} - \frac{2(1 - e^{-(a+b)t})}{(a+b)}\right]$
3-phase AC capacitor bank switching V = source voltage, R = resistance, L = inductance, C = shunt capacitance, all per phase. ω = ac angular frequency. *Derived quantities …* $XX = \omega L - 1/(\omega C)$, $Zss = \sqrt{(R^2 + XX^2)}$ $\varphi = \tan^{-1}(XX/R)$, $\alpha = R/2L$, $\omega_0 = \sqrt{(1/LC)}$ $a = 1 - \frac{\omega L \sin\varphi + \alpha L \cos\varphi}{Z_{ss}}$ $b = \frac{\omega_n L \cos\varphi}{Z_{ss}}$ $\beta = \tan^{-1}\frac{b}{a}$ $\gamma = \tan^{-1}\frac{\omega_n}{\alpha}$.	$i = \frac{\sqrt{2}V}{Z_{SS}}\left[\frac{Z_{SS}\sqrt{a^2 + b^2}}{\omega_n L} e^{-\alpha t} \sin(\omega_n t - \beta) + \cos(\omega t - \varphi)\right]$	$\left(\frac{\sqrt{2}V}{Z_{SS}}\right)^2 [C_1 + C_2 + C_3]$ where $C_1 = \frac{(a^2 + b^2)Z_{SS}^2}{4\omega_n^2 L^2}\left[\frac{1 - e^{-2\alpha t}}{\alpha} + \frac{e^{-2\alpha t}\cos(2\omega_n t - 2\beta + \gamma) - \cos(\gamma - 2\beta)}{\omega_0}\right]$ $C_2 = \frac{\omega t}{2} + \frac{\sin 2(\omega t - \varphi)}{4\omega} + \frac{\sin 2\varphi}{\omega}$ $Y = [\propto \sin(\beta + \varphi) - (\omega_N + \omega)\cos(\beta + \varphi)]\cos(\omega_n + \omega)t - [\propto \cos(\beta + \varphi) - (\omega_N + \omega)\sin(\beta + \varphi)]\sin(\omega_n + \omega)t$ $C_{3A} = \frac{Ye^{-\alpha t} - \alpha\sin(\beta + \varphi) + (\omega_n + \omega)\cos(\beta + \varphi)}{\alpha^2 + (\omega_n + \omega)^2}$ $Z = [\propto \sin(\beta - \varphi) - (\omega_N - \omega)\cos(\beta - \varphi)]\cos(\omega_n - \omega)t - [\propto \cos(\beta - \varphi) - (\omega_N - \omega)\sin(\beta - \varphi)]\sin(\omega_n - \omega)t$ $C_{3B} = \frac{Ze^{-\alpha t} - \alpha\sin(\beta - \varphi) + (\omega_n - \omega)\cos(\beta - \varphi)}{\alpha^2 + (\omega_n - \omega)^2}$ and then $C_3 = \frac{Z_{SS}\sqrt{a^2 + b^2}}{\omega_n L}(C_{3A} + C_{3B})$

Note 1. For DC capacitor charging the equations are the same as those for capacitor bank discharge, except that V now represents the DC source voltage

Note 2. In some cases when t is small the difference between two nearly equal numbers needs to be calculated, which can result in very large errors, even when using double precision arithmetic. In these cases the short-time formula must be used.