THE CALCULATION OF THE PARAMETERS OF THE PLASMA OF THE ARC IN A FUSE

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1. Abstract

Developed in the channel model approximation is a mathematical model of an electric arc in a quartz filled fuse with a foil strip fusible element. The arcing takes place in quartz (SiO_2) vapours. The model accounts for molecular dissociation of SiO_2 into SiO, Si and O, ionization of Si atoms, dissipation of energy from the arc channel by radiation and thermal conduction. The model allows to calculate the channel plasma temperature, arc channel thickness, axial electric field value and particles density during the whole arc extinction process from known values of current and pressure in the arc channel.

2. System of equations

When emergency currents are cut off by a fuse with strip fusible elements there appears an electric arc, the arcing taking place in a closed chamber within fulgurite. In the first approximation the chamber can be considered as a right-angle prism of width a, thickness 2 b and length L .

Within the scope of the channel model Namitokov and Frenckel[1] obtained a system of equations describing such a discharge

$$I = 2 a x_o \sigma(T_o) E \quad (1)$$

$$IE = \frac{2a}{b - x_o} \int_{T_w}^{T_o} \chi(T')dT' + 2 a x_o u(T_o) \quad (2)$$

$$\frac{[IE - 2a x_o u(T_o)]^2}{8a^2} = E^2 \int_{T_w}^{T_o} \chi(T')\sigma(T')dT' - \int_{T_w}^{T_o} \chi(T')u(T')dT' \quad (3)$$

Here I - arc current; E - axial electric field intensity in arc; $2X_o$ - thickness of arc channel; σ and T_o - conductivity and temperture of arc plasma, respectively; χ - thermal conduction of gas surrounding the channel; u - density of radiation losses; T_w - chamber wall temperature. The arc channel width is accepted to be equal to the chamber width.

The system of equations (1) - (3) allows to find the main dicharge parameters T_o, X_o and known values of the arc current and pressure (P) in the chamber. For further calculations it is necessary to know the dependence of quantities σ, χ and u on the plasma temperature and pressure.

To find these relationships we assume that in a quartz filled fuse the arcing takes place in quartz vapours, the ionized gas being silicon (Si) vapours, the chamber pressure reaches the value of few tens of atmospheres, and the plasma temperature reaches 20000 K. Such a discharge can be considered as a high-pressure electric arc and the sischarge plasma as strongly ionized. Marshak[2] has shown that in this case plasma conductivity is determined by the following expression

$$\sigma = 3.4\,10^{-6} \frac{T^{5/2}}{\varphi_i} = 4.05\,10^{-7} T^{5/2} \quad (4)$$

where φ_i is the ionization energy (for Si φ_i = 8.15 eV).

Raiser[3] has shown that in high-pressure arcs the main type of radiation within the considered temperature range is recombination radiation. Hence the radiation losses density is determined by the formula

$$u = 5.4\,10^{-35} \frac{N_e^2}{\sqrt{T_o}} \quad (5)$$

where N_e is the electrons density in plasma.

To find the thermal conduction we use the known expression for thermal conduction of monatomic gas

$$\chi = \chi_o \sqrt{T_o} \quad (6)$$

where χ_o is a constant found from experimental data.

The SiO_2 molecules which evaporize

from the chamber walls dissociate in the chamber space. The dissociation thereat takes place in two stages. At first, the reaction $SiO_2 \longrightarrow SiO + O$ takes place, the dissociation energy (φ_d) being 4.95 eV. Then the reaction $SiO \longrightarrow Si + O$ takes place with the dissociation energy equal to 7.8 eV. The latter value is close to the silicon ionization energy. Therefore, during calculation it should be taken into account that in the arc channel, besides Si and O atoms, there could be SiO molecules.

According to Dalton's law, the pressure in the chamber with account of the dissociation and ionization reactions, and in the assumption that the reaction $SiO_2 \longrightarrow SiO + O$ has fully taken place, is found from the equation

$$P = KT_o(N_{O1} + N_{SiO} + N_{Si} + N_{Si}^+ + N_{O2} + N_e) \quad (7)$$

where N_{O1} - density of oxygen atoms arising in the reaction $SiO_2 \longrightarrow SiO + O$; N_{SiO} - density of SiO molecules; N_{Si} - density of silicon atoms; N_{Si}^+ - density of Si ions; N_{O2} - density of oxygen atoms arising in the reaction $SiO \longrightarrow SiO + O$; N_e - density of electrons; K - Boltzmann's constant.

Equation (7) can be simplified. Let us introduce the designations: N_{SiO}^S - density of SiO molecules prior to their dissociation; N_{Si}^S - density of silicon atoms prior to their ionization.

Then, taking into account that

$$N_{Si} + N_{Si}^+ = N_{Si}^S; \quad N_{Si}^+ = N_e;$$

$$N_{SiO} + N_{O2} = N_{SiO}^S = N_{O1}$$

we obtain

$$P = KT_o(2N_{SiO}^S + N_{Si}^S + N_e) \quad (8)$$

We can ignore ionization of oxygen atoms bacause the oxygen ionization energy is significantly greater than the silicon atoms ionization energy.

The processes of dissociation and ionization in equilibrium isothermic plasma which is present in high-pressure arcs are described on the basis of the mass action law by the following equations:

$$\frac{N_{Si}^{S\,2}}{N_{SiO}^S - N_{Si}^S} = K_1(T_o) \quad (9)$$

$$\frac{N_e^2}{N_{Si}^S - N_e} = K_2(T_o) \quad (10)$$

where $K_1(T_o)$ and $K_2(T_o)$ are the dissociation and ionization constants, respectively.

According to Smirnov[4], the dissociation constant for diatomic molecules is of the form:

$$K_1(T_o) = \frac{g_{Si} g_o}{2 g_{SiO}} \times \frac{(1 - e^{-\frac{\hbar\omega}{KT_o}})}{2\pi r_o^2} \times \sqrt{\frac{\pi \mu K T_o}{h^2}} \; e^{-\frac{\varphi_d}{KT_o}} \quad (11)$$

where g_{Si}, g_o and g_{SiO} are the statistical weights of the silicon and oxygen atoms and SiO molecule; $\hbar\omega$ - distance between the molecule oscillation levels; r_o - equilibrium distance between atom nuclei in the molecule; h - Planck's constant; μ - reduced mass of atoms.

For SiO we have: $\mu = 1.69\ 10^{-26}$ kg; $\hbar\omega = 1241\ cm^{-1} = 0.154$ eV; $r_o = 1.51\ 10^{-10}$ m; $g_{Si} = g_o = 9$; $g_{SiO} = 2$. Substituting these values into (11), we obtain:

$$K_1(T_o) = 5.16\,10^{29}(1 - e^{-\frac{1790}{T_o}}) \times \sqrt{T_o}\, \exp(-\frac{9.048\,10^4}{T_o}) \quad (12)$$

Taking into account that the statistical weight of the silicon ion is equal to 6, the ionization constant takes the form:

$$K_2(T_o) = 3.23\,10^{21} T_o^{3/2} \exp(-\frac{9.454\,10^4}{T_o}) \quad (13)$$

Finding N_{SiO}^S from (9), and substituting its expression into (8), we obtain:

$$P = KT_o\left(\frac{2N_{Si}^{S\,2}}{K_1(T_o)} + 3N_{Si}^S + N_e\right) \quad (14)$$

Combining equations (1) - (3), (10) and (14) with expressions (4) - (6), (12) and (13), we obtain a system of equations allowing to find T_0, X_0, E, N_{Si}^{s} and N_e from I and P. But before starting to solve this system of equations we shall perform a number of simplifications.

3. Solving the system of equations

Preliminary calculations have shown that in the pressure range being considered, at small temperatures $N_{Si}^{s} \gg N_e$, hence the quantity N_e in the denominator of equation (10) and in the r.h. side of equation (14) can be ignored. Then, as the temperature rises, the quantity $2N_{Si}^{s2}/K_1(T_0)$ in (14) becomes small as compared to N_{Si}^{s}. As before, N_e thereat remains small. Thus, the whole temperature range can be divided into two regions. In the first region at low temperatures N_e can be ignored in the denominator of (10) and in the r.h. side of (14). Then from these equations we obtain:

$$N_{Si}^{s} = -\frac{3}{4}K_1(T_0) + \sqrt{\frac{9}{16}K_1^2(T_0) + \frac{PK_1(T_0)}{2KT_0}} \qquad (15)$$

$$N_e^2 = -\frac{3}{4}K_1(T_0)K_2(T_0) + K_2(T_0)\sqrt{\frac{9}{16}K_1^2(T_0) - \frac{PK_1(T_0)}{2KT_0}} \qquad (16)$$

Substituting (16) into (2), (3) and (5), and computing the integral in (3) with account of $T_0 \gg T_w$, from the system of equations (1) - (3) we obtain:

$$I = 2\alpha x_0 \sigma_0 T_0^{5/2} E \qquad (17)$$

$$IE = \frac{4\alpha X_0 T_0^{3/2}}{3(\delta - X_0)} - 8.1\,10^{-25}\frac{\alpha x_0}{\sqrt{T_0}} \times K_1(T_0)K_2(T_0) + 1.08\,10^{-34}\frac{\alpha x_0}{\sqrt{T_0}} \times K_2(T_0)\sqrt{\frac{9}{16}K_1(T_0) - \frac{PK_1(T_0)}{2KT_0}} \qquad (18)$$

$$\frac{2X_0T_0^2}{9(\delta - X_0)^2} = 1\,10^{-x}E^2T_0^3 + 4.05\,10^{-35} \times \frac{K_1(T_0)K_2(T_0)T_0^2}{\varphi_i + \varphi_d} - \frac{4.05\,10^{-35}K_2(T_0)}{2K_1(T_0)\left(1 + \frac{\varphi_d}{T_0}\right)} \times \left\{\left[K_1(T_0) + \frac{4P}{3KT_0}\right]\sqrt{K_1^2(T_0) + \frac{8K_1(T_0)P}{3KT_0}} - \frac{32P^2}{9K^2T_0^2}\ln\left[\sqrt{1 + \frac{3KT_0K_1(T_0)}{8P}} + \sqrt{\frac{3KT_0K_1(T_0)}{8P}}\right]\right\} \qquad (19)$$

In the second temperature region we ignore the term $2N_{Si}^{s2}/K_1(T_0)$ in the r.h. side of (14). In this case, from (10) and (14) we obtain

$$N_e = \frac{2}{3}K_2(T_0)\left[\sqrt{1 + \frac{3P}{4K_2(T_0)KT_0}} - 1\right] \qquad (20)$$

$$N_{Si}^{s} = N_e + \frac{N_e^2}{K_2(T_0)} \qquad (21)$$

Using (20) in the same way as (16), and performing the same transformations, from (2) and (3) we obtain:

$$IE = \frac{4\alpha X_0 T_0^{3/2}}{3(\delta - X_0)} + 4.8\,10^{-35}\frac{\alpha x_0}{\sqrt{T_0}} \times K_2^2(T_0)\left[\sqrt{1 + \frac{3P}{4K_2(T_0)KT_0}} - 1\right]^2 \qquad (22)$$

$$\frac{2X_0T_0^2}{9(\delta - X_0)^2} = 1\cdot 10^{-x}E^2T_0^3 - 2.4\,10^{-35}\frac{T_0}{\varphi_i}K_2^2(T_0) +$$

$$+\frac{2.4\,10^{-35}}{1.5+\frac{\varphi_i}{T_0}}\left\{\left[K_2(T_0)+\frac{3P}{8KT_0}\right]\times\right.$$

$$\times\sqrt{K_2(T_0)+\frac{3P}{4KT_0}-\frac{9P^2}{32K^2T_0^2}}\;\times$$

$$\times\ln\left[\sqrt{\frac{4K_2(T_0)KT_0}{3P}}\;+\right.$$

$$\left.\left.+\sqrt{1+\frac{4K_2(T_0)KT_0}{3P}}\right]\right\}-$$

$$-1.8\,10^{-35}\frac{PK_2(T_0)}{K\varphi_i}\left(1-\frac{5}{2}\frac{T_0}{\varphi_i}+\right.$$

$$\left.+\frac{35}{4}\frac{T_0^2}{\varphi_i^2}\right) \qquad (23)$$

The third equation of this system is equation (17).

The boundary between two temperature regions is determined conventionally by the inequality

$$\frac{2N_{Si}^S}{3K_1(T_0)}>0.05 \qquad (24)$$

This inequality means that the boundary between the first and second temperature regions passes through the temperature at which the first summand in the r.h. side of (14) is 20 times smaller than the second one. Substituting (15) into (24) we obtain

$$0.5\sqrt{1-\frac{8P}{9KT_0K_1(T_0)}}-0.55>0 \qquad (25)$$

In the first temperature region this inequality is satisfied, and in the second one it is not.

The obtained systems of equations are solved on a PC.

If the solution is obtained in the first temperature region, then besides quantities T_0, X_0, E, and we can find

$$N_{Si}=N_{Si}^S-N_e$$

$$N_{Si0}=\frac{N_{Si}^{S2}}{K_1(T_0)}$$

$$N_0=N_{Si0}+2N_{Si}^S \qquad (26)$$

If the solution is obtained in the second temperature region, then additionally we can find

$$N_{Si}=N_{Si}^S-N_e,\quad N_0=2N_{Si}^S \qquad (27)$$

Besides, knowing the arc voltage (U_d), we can find the arc length

$$L=\frac{U_d-U_{ac}}{E} \qquad (28)$$

Here U_{ac} is the potential drop in the electrode vicinity. Namitokov and Frenckel[5] have shown that for fusible elements Al, Cu and Ag U_{ac} = 40 V.

The value of X_0 was chosen by comparing the calculated and experimental values of L . For aluminum X_0 = 1 10^{-3} mkg/s^{3}K$^{3/2}$; for copper and silver X_0 = 1.6 10^{-3} mkg/s^{3}K$^{3/2}$.

4. Calculation results

The obtained model was used to calculate the plasma parameters of an arc discharge in a fuse according to Namitokov's and Frenckel's[6,7,8] experimental data.

Shown in Fig. 1 are the results of calculating the plasma temperature, axial electric field intensity and burn-out length of a fuse with a copper fusible element 10 mm wide, 0.14 mm thick and one rectangular neck in the centre 1 mm long and 1 mm wide. Tests were carried out in a d.c. circuit with voltage 500 V, anticipated current 1400 A, time constant 2.3 ms. The whole process lasted for 71 ms. Calculations began in 4.4 ms from the moment of arc emergence. As seen from the given calculations, at the initial stage of the arc extinction process the plasma temperature is exceedingly high and reaches 18 10^3 K. This corresponds to values obtained by spectroscopy measurements. Further the plasma temperature drops, but even at the end of the arc extinction

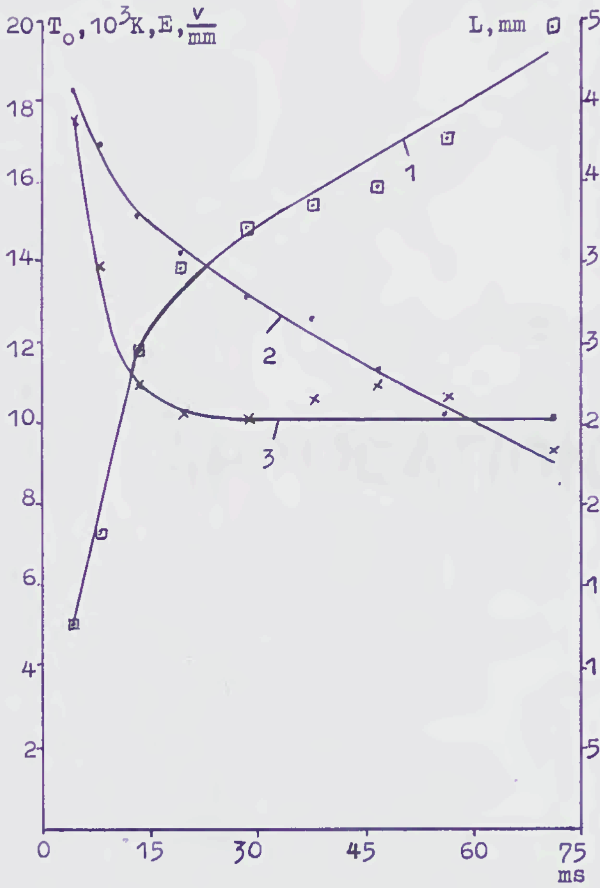

Fig.1 1 – L; 2 – T_o; 3 – E

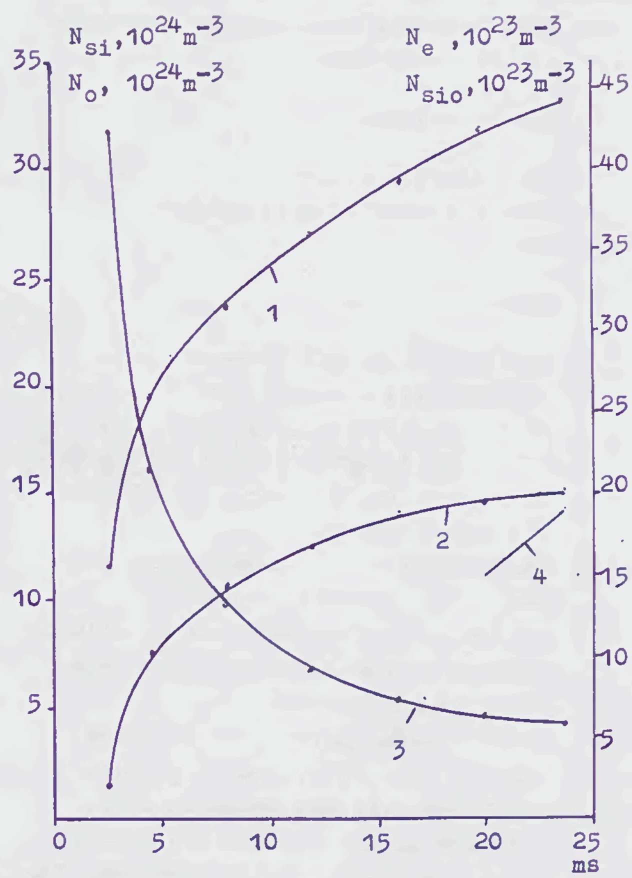

Fig.2 1 – N_o; 2 – N_{si}; 3 – N_e; 4 – N_{sio}

process the plasma remains strongly ionized. The presence of SiO molecules can be ignored during the whole process. The behaviour of the axial electric field intensity value corresponds to that obtained experimentally by Ranjan R. and Barrault M.[9] At the initial stage the fusible element burn-out process is intensive, and then it slows down. The final burn-out length value obtained experimentally is 44 mm, and the calculated value is 49.5mm.

Shown in Fig. 2 are the results of calculating the particles density in the arc plasma of a fuse with a silver fusible element 2.5 mm wide and 0.14 mm thick with one neck on the centre 1 mm long and 1 mm wide. Experiments were carried out with d.c. voltage 500 V, anticipated current 3000 A, time constant 5.1 ms. Arcing time was 23.6 ms. These experiments were distinguished by very high pressure in the fuse arc channel. Thus. measurements have shown that at the end of the arc extinction process the pressure in the arc channel reaches 56.5 10^5 Pa. This accounts for the high particles density in the arc channel. At the final stage of the process it is necessary to take into account SiO molecules, and calculations are carried out in the first temperature region. At the early stages of arc extinction calculations are carried out in the second temperature region. The final burn-out length value obtained experimentally is 46 mm, and the calculated value is 47.4 mm.

5. Conclusion

The developed mathematical model of an electric arc in a quartz filled fuse allows to calculate the discharge parameters of gas-discharge plasma during the whole arc extinction process. Checking the behaviour of the obtained relationships requires experimental determination of these parameters. Calculation of discharge parameters at the end of the arc extinction process shows satisfactory agreement with experimental data.

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Session 7

APPLICATIONS AND TRENDS