

# THERMAL MODEL OF THE EVOLUTION OF FRAGMENTS INSIDE A MICROSCOPIC SPOT: A MULTISCALE APPROACH OF THE INTERACTION PLASMA/CATHODE

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**Abstract:** In this paper we propose an explanation for the great dispersion (several orders of magnitude) which exists in literature for the various values of essential parameters (such as surface power density or current density) of influence in the power balance at the surface of a copper cathode submitted to an electric arc. In this work this dispersion is explained by the difference in the space and time scales considered in the different works published. A simple model is proposed which constitutes a first step allowing to explain the difference in the values for the surface power density brought to the cathode surface.

**Keywords:** electric arc, cathode spot, multi-scale.

## 1. Introduction

For many applications (switching devices, plasma torches, study of lightning impact, arc welding...) the study of the electrode erosion under the action of an electric arc is of great importance. A great deal of literature concerning electrode erosion mechanisms for various experimental conditions is available. Many parameters are liable to influence the erosion phenomena. Among them, we can quote: the arc current intensity, the arcing time, the electrode material and polarity, the electrode gap and shape, the nature of the covering gas, the arc mobility on the electrodes... In these conditions, a theoretical predictive evaluation of the erosion caused by the electric arc action in given experimental conditions remains a very difficult work.

Another way to tackle the problem of electrode erosion is the assessment of the power balance at the electrodes. Many works concerning power balance at the cathode surface have already been published. However, in the case of low melting temperature electrodes (copper cathode for instance), there exists a great scattering (several orders of magnitude) in the values of several essential parameters such as the current density at the cathode surface or the surface

power density brought by the electric arc to the cathode surface.

The aim of his paper is to propose a possible explanation of the parameter value scattering. The root idea of this explanation is relative to different observations or modelling scales used for space as well as for time description.

In the first part of this paper the description of the power balance at the cathode is briefly recalled as well as the different values proposed in the literature.

In the second part of the paper a “multi-scale” approach of the cathode heating is proposed. This approach is based on the work of Jüttner [1] who observed different structures on the cathode surface. First results are then presented which may constitute a first step to explain the scattering which exists in the literature concerning the surface power density value brought by the arc to the electrode.

To finish, perspectives and remaining questions are presented.

## 2. Reminder of the power balance at the cathode and the various spot structures observed

The power flux brought by the arc to the cathode has several origins:

- the electrons (emitted or back scattered). The power « brought » by the emitted electrons to the cathode surface is usually written under the following form ( $Q_{ee}$ ):

$$Q_{ee} = J_e \cdot F_N$$

where  $J_e$  is the emitted electron current density. For a given cathode material,  $J_e$  is a function of the macroscopic field in front of the cathode surface and of the surface temperature.  $F_N$  is called Nottingham potential. Several analytical expressions exist for  $J_e$  or  $F_N$ , which are obtained by making approximations according to the temperature and electric field range concerned [2,3]. Numerical calculations have also been carried out [4, 5].

- the ions impinging the surface, which bring to the cathode a fraction of their kinetic energy and which may bring also some energy provided by neutralisation phenomena that occur near the surface. Usually this power flux (denoted here  $Q_i$ ) is written under the following form [6]:

$$Q_i = (1 - \alpha)(\varepsilon U_c + V_i - \phi_0)J$$

where  $\alpha$  is the fraction of electron current density,  $\varepsilon$  is the fraction of kinetic ion energy transferred to the surface,  $U_c$  is the near-cathode voltage drop,  $V_i$  is the ionisation potential,  $\phi_0$  is the work function of the cathode material and  $J$  is the current density in the cathode root.

- the neutral species which bring a fraction of their kinetic energy to the surface and may also bring energy provided by excitation, ionisation phenomena that occur when they interact with the surface.

- the plasma radiation and the cathode surface radiation

- the various chemical reactions at the cathode surface

- the cathode Joule heating

In the case of “cold” cathodes (Cu, Ag, Ni, ...), many works have been done to assess essential parameters like the current density in the cathode spot and/or the surface power density brought by the arc to the cathode surface (these two parameters are linked). The proposed value ranges are quite large.

Concerning the current density determination, several experimental methods have been proposed: the observation of the tracks left by the arc root, the observation of the luminous zone over the cathode surface, spectroscopic study (Zeeman effect), the use of coils in the cathode, the measurement of the force exerted by the arc on the electrode...

All these studies lead in the case of « static » or moving arc to current density values in the range  $10^8 - 10^{12}$  A/m<sup>2</sup> [7 to 13]. Moreover, modelling works [14 to 18] proposed values in the range  $5 \cdot 10^9 - 5 \cdot 10^{11}$  A/m<sup>2</sup>.

Concerning the surface power density modelling works proposed values ranging from  $10^{10}$  W/m<sup>2</sup> to  $10^{12}$  W/m<sup>2</sup>. Several experimental works [16] and [19 to 22] based on the observation of tracks, on the measurement of the mass losses of the electrodes, on the measurement of the energy dissipated in the electrodes have lead to values of the surface power density in the range  $5 \cdot 10^9$  W/m<sup>2</sup> to  $8 \cdot 10^{11}$  W/m<sup>2</sup>.

As we can see there is a great scattering in the proposed values of these two parameters. The aim of this paper is then to try to explain this scattering. For

that, the root idea is that it may be due to the different space and time scales considered to describe the arc-electrode interactions. Indeed several spot structures have been observed and described in numerous works [1], [23 to 25]. In [1] Jüttner has proposed a classification of these structures through different parameters like the life duration, the size (diameter) and the intensity in the spot. It may be summed up as follows with the help of three levels of structures (according to Jüttner):

- Structures of level A: Cathode spots characterized by a current intensity around 50 - 100 A, diameters around 50 - 100  $\mu$ m and a life duration about some tens of  $\mu$ s.
- Structures of level B: Fragments or microspots having diameters about 20  $\mu$ m, current about 20A and a life duration about 10 - 20 ns.
- Structures of level C: Subfragments or cells characterized by a current smaller than 5 A and diameters smaller than 5  $\mu$ m.

Figure 1 from Jüttner [1] shows an example of spot fragments (observed with the help of a high speed camera). According to Jüttner, the irregular structure indicates the presence of cells.

Other larger structures are also observed as soon as the arc current intensity or the arc duration increases. For instance figure 2a and 2b show macroscopic craters having an axi-symmetric geometry. They have been observed with the help of a 3D optical profiler. In the first case the copper cathode was submitted to an electric arc of 300 A with an arc duration of 300  $\mu$ s, in the second case the arc duration was equal to 5 ms and the mean current intensity to 100 A. In figure 2a, the crater diameter is about 1 mm and macroscopic waves of solidified liquid metal may be noticed at the crater edge. In these two examples, the crater shapes allow to think that they result from the action of a unique static heat flux having an axi-symmetric distribution on the cathode surface and the action of multiple fragments or small spots is not detectable on such tracks.

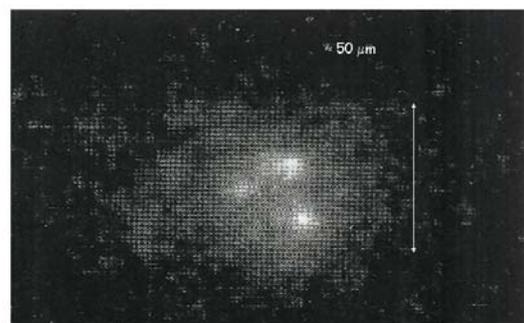
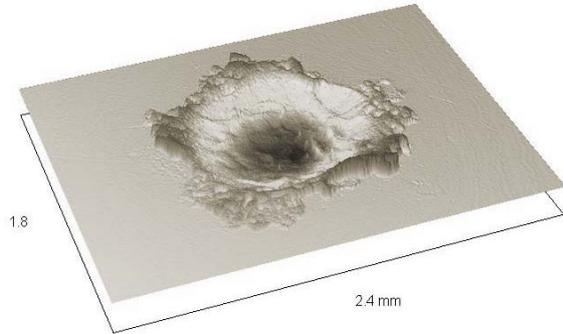


Figure 1: High speed photography of fragments (Jüttner [1])

For higher arc current intensities macroscopic liquid metal “bath” may be observed as well as macroscopic liquid metal droplets. Figure 3 from Devautour [13] shows macroscopic liquid copper droplets ejected from the cathode surface. In this case the arc intensity was equal to 1000 A the arc duration



to 5 ms.

Figure 2a: Macroscopic crater observed on a copper electrode. The arc duration is equal to 300  $\mu$ s and the current intensity is about 300 A.

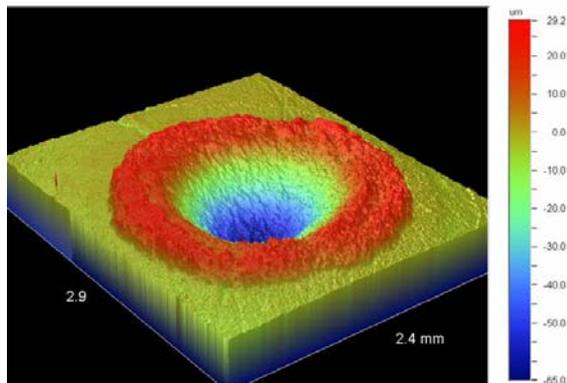


Figure 2b: Macroscopic crater observed on a copper electrode. The arc duration is equal to 5 ms and the current intensity is about 100 A.

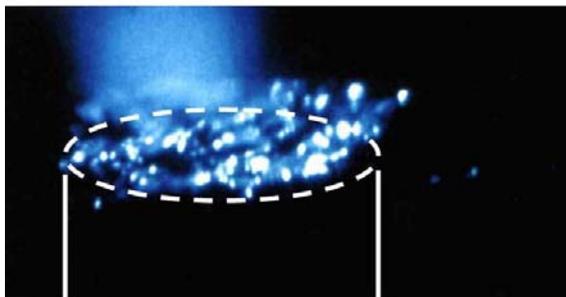


Figure 3: Copper electrode – Macroscopic droplet ejection under the arc action.

A question may be asked:

- How a set of microscopic structures like fragments may create tracks as the one observed in figure 2, that is to say a crater of 1 mm diameter having an axi-symmetric shape and showing a macroscopic molten zone and macroscopic molten waves? How may the connection between the various spot structures be done?

### 3. Proposition of a multi-scale approach

The aim of this paper is to find an equivalence between:

- the heating of an electrode under the action of multiple very intense mobile heat fluxes applied during a very short time

and

- the heating of an electrode by a static heat flux

The objective is also to find if it is possible to obtain crater shapes similar to the one presented in figure 2 under the action of numerous short duration mobile fragments and what is the relation between heat flux characteristics of each fragment and the characteristics of a static heat flux susceptible to cause crater of figure 2.

#### 3.1 The transition from the fragment to the cathode spot (structure B to structure A)

The aim of the first step presented here is to find a relation between the heating of several fragments (structures of level B) and spot heating (structure of level A).

For that we have simulated the heating of five fragments of 10 A, with a life duration of about 20 ns for the whole life duration of a cathode spot of 50 A, that is to say 20  $\mu$ s. The electrode is a copper cylinder (8mm in diameter)

Fragments are unstable fluctuating structures [1]. According Jüttner [1], the spatial fluctuation of the fragments does not mean a real movement: one fragment dies and another is ignited at a different location. As a consequence the centre of gravity of the cathode spot constituted of the five fragments is displaced. Jüttner has described this displacement with the help of a random walk law. The probability  $W(R)dR$  to have a displacement in the interval  $(R, R+dR)$  irrespectively of the direction is given by :

$$W(R)dR = \frac{2R}{\langle R^2 \rangle} \exp\left(\frac{-R^2}{\langle R^2 \rangle}\right) dR \quad (1)$$

where  $\langle R^2 \rangle = 4Dt$  is the mean value during a measuring interval  $t$ .  $D$  is the diffusion parameter.

Several authors have proposed values for D near  $2 \cdot 10^{-3} \text{ m}^2/\text{s}$ .

### 3.1.1 Modelling of the heating – Description of the numerical modelling of the electrode heating

In this part a brief description of the numerical model used to simulate the electrode heating under the set of fragments is given.

The main properties and boundary conditions of the model are the following:

- (i) The electrode is a copper cylinder of 8 mm diameter.
- (ii) At the beginning of the heating the fragments are disposed randomly around the spot gravity centre. An order of magnitude of the distance between the spot centre and each fragment centre has been given in [1] and is about  $25 \mu\text{m}$ . Each fragment has a life duration of 10 - 20 ns and the spot centre moves according to the law given by equation (1).
- (iii) The cathode material (copper) properties: specific heat and thermal conductivity are taken to be temperature dependent.
- (iv) The density has been taken constant. This approximation is not very restrictive because for metals like copper, the thermal dilatation coefficient is small.
- (v) Joule heating has been neglected. This approximation is valid as far as the current density values (denoted J) remain smaller than  $10^{11} \text{ A/m}^2$  as it has been shown by He et al.[15] for a flat electrode
- (vi) The motions induced in the molten part of the electrode by Lorentz forces, Marangoni effects or plasma jets/electrode surface interactions, etc... are neglected.
- (vii) The heat flux on the lateral sides of the electrode has been neglected. This assumption is valid because we consider short duration electric arcs.
- (viii) The bottom temperature is fixed to  $T = T_0 (=25^\circ\text{C})$  on the lower cross section of the electrode. This is justified if the electrode is tall enough (some millimetres) and if the arc duration is short enough to avoid the heat to diffuse to the electrode bottom during the arc.
- (ix) During its life duration each fragment brings an heat flux characterized by the power P(t) and its surface power density Q(r,t). The values taken for the power and the power density are discussed hereafter.
- (x) Mathematical formulation of the problem: If we neglect the liquid motion in

the molten zone, we have then to solve the following equations:

(1) The heat diffusion equation in the non-stationary regime in each phase (solid and liquid) which takes the following form :

$$\rho C_p(T) \frac{\partial T}{\partial t} = \text{div}(k(T) \vec{\nabla} T) + S \quad (2)$$

Where:  $\rho$  is the material volumetric mass,  $C_p$  the material specific heat, T the temperature, t the time, k the material thermal conductivity, S may represent some power density sources (Joule effect,...).

(2) The equation at the boundary between each phase which has the following form :

$$[k \nabla T]_i^2 = -\rho L_{1 \rightarrow 2} v_s \quad (3)$$

where  $v_s$  is the velocity of the boundary between two phases and  $L_{1 \rightarrow 2}$  is the latent heat of the change of state (state 1  $\rightarrow$  state 2). This second condition is called ‘‘Stefan condition’’. Several numerical methods exist in order to simulate the melting front’s evolution [26] which require to track the melting front at each time step to apply equation (2). For that several authors have proposed methods using variable time or space steps. In fact the difficulty of the ‘‘Stefan problem’’ lies in the discontinuity of the enthalpy function H(T). The method we used overcomes this difficulty by making the enthalpy function continuous. This function is schematically described in figure 4.

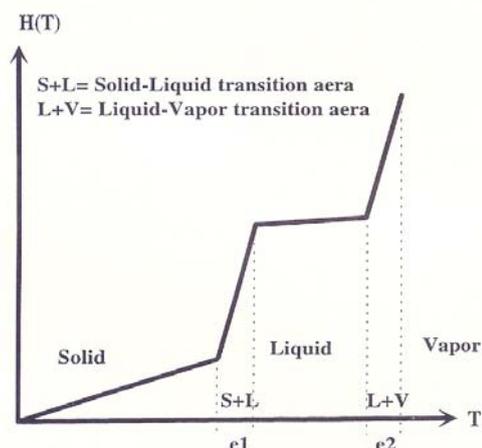


Figure 4: Schematic representation of the continuous enthalpy function used to treat the phase change problem.

The phase changes are continuously realized on a temperature interval  $[T_c - \epsilon ; T_c + \epsilon]$ ,

where  $T_c$  may be the melting or the vaporization temperature. In fact, such a method consists of creating a fictitious specific heat equal to  $L/2\varepsilon$ . Then we have only to solve equation (1). The determination of the boundary between two phases is then realized *a posteriori* after the calculation of the temperature distribution. The use of a transition zone introduces an uncertainty in the determination of the liquid or vapour amount created during the heating. However, we have noted that as soon as  $\varepsilon$  is lower than 20 °C the  $\varepsilon$  value has no real influence on these amounts. In our calculations we have taken  $\varepsilon = 5^\circ\text{C}$ .

Concerning the problem relative to the free moving vapour front, the method used has already been described in [27] and [28].

(xi) The discretization of the heat equation is based on a finite-difference numerical method with an implicit time scheme. The discretization time has been taken between 0.1 and 1 ns (according to Q value). The space discretization is not uniform: a high density mesh just under the electrode surface which evolves and becomes larger at the electrode bottom. The smallest values of the space discretization depend on Q and are included between 0.1 and 5  $\mu\text{m}$ .

### 3.1.2 Example of results of multi fragment calculations.

The values chosen to characterize the power flux brought by a fragment are the following:

- the intensity in each fragment is taken equal to 10 A.
- The current density J in the fragment is equal to  $7 \cdot 10^{10}$  A/m<sup>2</sup> and the surface power density Q equal to  $3 \cdot 10^{11}$  W/m<sup>2</sup>. These values correspond to results of different modelling works [17] or [18] concerning the fragment description.
- The arc duration is equal to 20  $\mu\text{s}$ .
- The life duration of each fragment has been taken equal to 20 ns.

In figure 5 we have plotted the different positions of the various fragment centres and the shape of the total heated area.

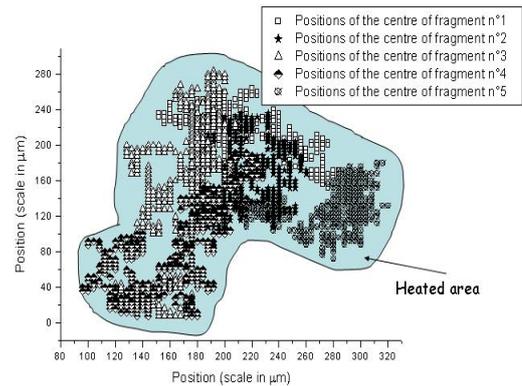


Figure 5: Example of the different positions of the various fragment centres and the shape of the total heated area.

Figure 6 is a photography of the track left by an arc of similar duration. We can note a qualitative correspondence between the two “tracks”

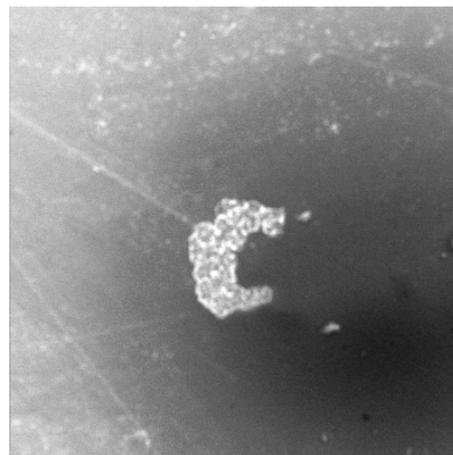


Figure 6: Photography of the track left by an electric arc (duration about 20  $\mu\text{s}$ )

Numerous calculations are required in order to obtain significant information concerning the heating due to five fragments. We have realized 13 calculations (computation times are quite important). In figures 7 and 8 we present the different values obtained concerning:

- the maximal molten depth value
- the mean surface extension of the molten zone. The molten zone is not a symmetric crater however it is possible to reach an assessment of a characteristic spatial extension.

Concerning the maximal molten depth we have obtained a mean value of about 24.5  $\mu\text{m}$  and 127.5

$\mu\text{m}$  for the mean value of the surface extension of the molten zone.

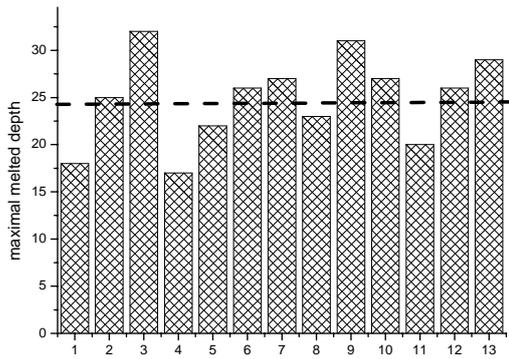


Figure 7: Maximal molten depth for 13 different calculations

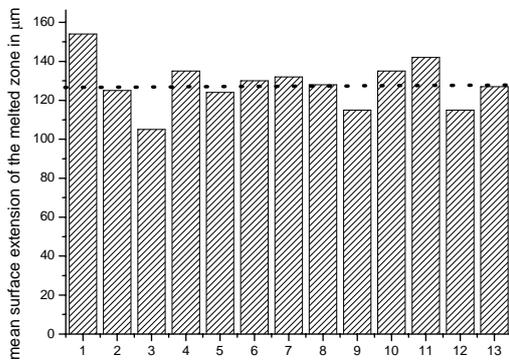


Figure 8: Mean surface extension of the molten zone for 13 different calculations

### 3.1.3 The research of an equivalent motionless unique heat flux.

The aim of this part is to answer to the following question: is it possible to find a heat flux able to create a “similar” heating to the one created by the previous five mobile fragments?

To define what we mean by “similar” heating we take comparison criterions which correspond to observable phenomena. In this case we have taken the molten depth and the surface extension of the molten zone.

We consider an unique motionless heat flux corresponding to an arc spot of 50 A. Its duration life is equal to 20  $\mu\text{s}$ . The problem is then axi-symmetric.

We calculate the molten depth and the molten radius for various values of the surface power density and of the current density.

#### Presentation of the results:

$J$  is the current density in the cathode spot and  $Q$  the surface power density brought to the electrode. In figures 9,10, 11 and 12 we have plotted the evolution of the molten depth and of the molten diameter versus the current density in the cathode spot for  $Q = 10^{10}$ ,  $3 \cdot 10^{10}$ ,  $5 \cdot 10^{10}$  and  $10^{11}$   $\text{W}/\text{m}^2$  respectively. We

have also recalled the value of 24.5 and 127.5  $\mu\text{m}$  obtained with the multi-fragments calculation for the mean depth and surface extension respectively.

- For  $Q \leq 5 \cdot 10^9 \text{ W}/\text{m}^2$  we do not observe melting for  $J$  in the range  $10^8 - 10^{11} \text{ A}/\text{m}^2$
- For  $Q = 10^{10} \text{ W}/\text{m}^2$  the molten depth value of 24.5 is never reached when  $J$  varies.
- For  $Q = 3 \cdot 10^{10} \text{ W}/\text{m}^2$  the molten depth reaches the value of 24.5  $\mu\text{m}$  when the molten diameter is practically two times greater than 127.5  $\mu\text{m}$ .
- For  $Q = 5 \cdot 10^{10} \text{ W}/\text{m}^2$  the two parameters (depth and diameter) are simultaneously near the two values (24.5 and 127.5  $\mu\text{m}$ ) obtained with the five fragments for  $J \approx 5 \cdot 10^9 \text{ A}/\text{m}^2$
- For  $Q = 10^{11} \text{ W}/\text{m}^2$  the molten depth is near 24.5  $\mu\text{m}$  whereas the molten diameter is only equal to 65  $\mu\text{m}$ .

The couple  $Q = 5 \cdot 10^{10} \text{ W}/\text{m}^2$  and  $J = 5 \cdot 10^9 \text{ A}/\text{m}^2$  then allows to obtain a heating very similar to the one obtained with the five mobile fragments (with  $Q = 3 \cdot 10^{11} \text{ W}/\text{m}^2$  and  $J = 7 \cdot 10^{10} \text{ A}/\text{m}^2$ ).

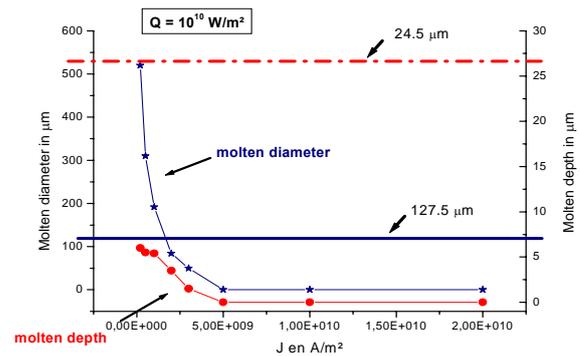


Figure 9: Evolution of the molten depth and of the molten diameter versus the current density in the cathode spot for  $Q = 10^{10} \text{ W}/\text{m}^2$

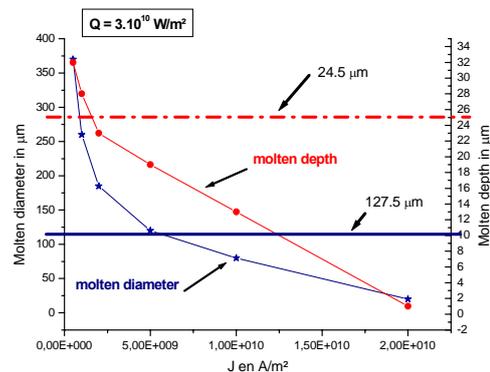


Figure 10: Evolution of the molten depth and of the molten diameter versus the current density in the cathode spot for  $Q = 3 \cdot 10^{10} \text{ W}/\text{m}^2$

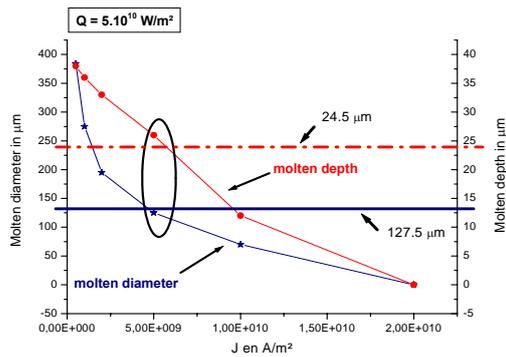


Figure 11 : Evolution of the molten depth and of the molten diameter versus the current density in the cathode spot for  $Q = 5 \cdot 10^{10} \text{ W/m}^2$

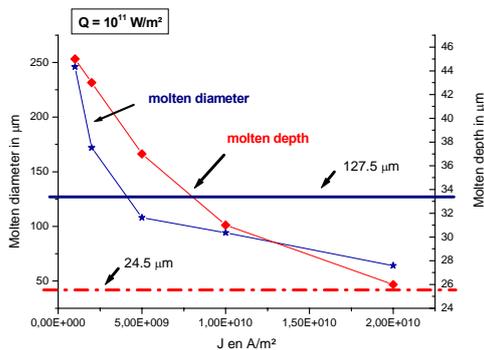


Figure 12 : Evolution of the molten depth and of the molten diameter versus the current density in the cathode spot for  $Q = 10^{11} \text{ W/m}^2$

#### 4 Conclusion, comments and questions

It is possible to consider that the action of several fragments (structure B according to Jüttner [1]) may be equivalent to the action of structures having the characteristics of a cathode spot (structure A [1]). The condition to achieve this equivalence is that the current density in the cathode spot is practically one order of magnitude smaller than the one in the fragment. That may contribute to explain a part of the scattering in the current density values proposed in literature.

However the “transition” to larger structures as the one presented in figure 2 (diameter  $\approx 1 \text{ mm}$ ) has still to be done. Moreover, the existence of a macroscopic liquid metal “bath”, or of macroscopic liquid metal droplets, remains particularly difficult to explain with the sole consideration of fragments.

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