STEADY THERMAL PHENOMENA IN FUSES WITH VARIABLE CROSS-SECTION

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INTRODUCTION The basic and active element of the electrical fuses being the fuse-link, the paper deals with the calculation of its overtemperature, taking into account only the temperature variation along its length.

The differential equations of variable cross-section fuse-links heating have been derived for the various forms of constrictions used in manufacturing electrical fuses. The types of constrictions that have been studied, resulted from performing some round, rhombic, and rectangular-shape holes.

The differential equations obtained for the steady-state were solved both analytically and by means of computers. By means of the found relations, the heating of fuse-links with several constrictions and with a single constriction has been calculated.

FUSE-LINK HEATING DIFFERENTIAL EQUATIONS. The heating of the fuse-links as well as the heating of the whole electrical fuse is due to the Joule effect of the electric current passing through the fuse-link. When establishing the differential equations we consider only the longitudinal heat transfer by thermal conduction, neglecting the radial transfer.

The General Differential Equation. The variable section fuse-link heating is established from the thermal energy balance of the elementary volume (dv) of the fuse-link. The calculations made in the paper [1] have led to the following differential equation.

$$\gamma c \frac{\partial\theta(x,t)}{\partial t} = \lambda \frac{\partial^2\theta(x,t)}{\partial x^2} + \rho(\theta) J^2(x,t) - K(\theta)\frac{l_x}{A_x}\left[\theta(x,t) - \theta_a\right] \qquad (1)$$

where: γ - is the bulk weight, in g/cm^3; c - specific heat, in ws/^{o}C;g.; λ - the thermal conduction coefficient, in W/cm^{o}C; ρ - fuse-link resistivity, in Ωcm; J - current density, in A/cm^2; K - overall heat-transfer coefficient, in W/cm^2 ^{o}C; l_x - circumference, in cm; A_x - cross-section, in cm^2; θ - temperature, in ^{o}C; θ_a - ambient temperature, in ^{o}C.

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The γ, c, λ, K and ρ parameters are depending on temperature. The γ, c, and λ values, varying a little with the temperature are considered to be constant. The resistivity variation with the temperature is important and can be viewed as a parabolic [2] or linear variation. The difference between these curves being a small one, in order to simplify things, the linear form is selected. The K heat-transfer coefficient, too, varies with the temperature. Certain experiments on the electrical fuses led to the conclusion that a parabolic function is the most accurate expression of its variation [3] , while K does not increase to a higher value than 1.5, (within the range of the studied temperatures), as compared to ρ , which increases about 3-4 times [2] .

<u>Constant Section Fuse-Link Heating Differential Equation, in Steady State</u> There are two cases that are interesting from a practical point of view. Round section fuse-links, of a <u>d</u> diameter, and rectangular section fuse-links, the rectangle sides being <u>b</u> and <u>g</u> . As far as the long fuse-links are concerned, relative to which the heat transfer by convection is also taken into account ($K \neq 0$),the steady state heating differential equation, derived from (1), is:

$$\lambda \frac{d^2\tau(x)}{dx^2}+\left[\alpha\rho_0 J^2-K\frac{l}{A}\right]\tau(x)=-\rho_a J^2 \qquad (2)$$

where: τ - overtemperature$(\theta-\theta_a)$ in $^\circ$C; ρ_0 - resistivity at $^\circ$C, in Ω cm; α - the coefficient of the resistivity variation with the temperature, in 1/$^\circ$C;ρ_a - the resistivity at the ambient temperature, in Ω cm; l/A - a constant equal to 4/d when considering the round section fuse-links, and 2/g when considering the rectangular section fuse-links

In the case of the constant cross-section fuse-link constrictions, (see Fig 1 a), by neglecting the heat transfer by convection (K = 0); the following heating differential equation is obtained from (2).

$$\frac{d^2\tau}{dx^2}+\frac{\alpha\rho_0}{\lambda}J^2\tau(x)=-\frac{\rho_a}{\lambda}J^2 \qquad (3)$$

<u>Variable Section Fuse-Links Heating Differential Equation, in Steady State</u> In Fig.1 the shape of the constrictions that are being studied, is presented. For the constrictions given in Fig.1b and 1c, the heating equations are obtained from (1) where lx/Ax and J(x) are replaced by the values corresponding to these constrictions. Taking into account the fact that the equation (1) refers to the steady-state, then $\frac{\partial\theta(x,t)}{\partial t}=0$

In the case of the constriction of Fig. 1b, derived from (1), by taking into account lx/Ax and J(x), and neglecting the fuse-link thickness relative to its width, the heating equation is.

$$\lambda\frac{d^2\theta(x)}{dx^2}+\frac{\rho_0 I^2\left[1+\alpha\theta(x)\right]}{4g^2\left[a+\frac{x}{x_1}\left(\frac{b}{2}-a\right)\right]^2}-\frac{2K}{g}\theta(x)+\frac{2K}{g}\theta_a=0 \qquad (4)$$

where:

$$\frac{l_x}{A_x}=\frac{2}{g}\ ;\ J(x)=\frac{I}{2g\left[a+\frac{x}{x_1}\left(\frac{b}{2}-a\right)\right]} \qquad (5)$$

Similarly, for the constriction of Fig. 1c, one obtaines:

$$\lambda \frac{d^2\theta(x)}{dx^2} + \frac{\rho_0 I^2\left[1+\alpha\theta(x)\right]}{4g^2\left[a+r-\sqrt{r^2-x^2}\right]} - \frac{2K}{g}\theta(x) + \frac{2K}{g}\theta_a = 0 \tag{6}$$

where:

$$\frac{lx}{Ax} = \frac{2}{g}\ ;\ J(x) = \frac{I}{2g\left[r+a-\sqrt{r^2-x^2}\right]} \tag{7}$$

FUSE-LINK HEATING EQUATIONS. In order to determine the solutions of the above given differential equations, a particular limiting-condition has been introduced, having the form:

$$-\lambda \frac{\partial \tau(x)}{\partial x}\bigg|_{x=0} = \eta\, \tau(x)\Big|_{x=0} \tag{8}$$

Where η is the heat-transfer coefficient, at the fuse-links ends. The fact has been accepted that the relation (8) is also valid for the coordinate corresponding to the end of the fuse-link constriction (x_1 coordinate, in Fig.1).

Constant Section Fuse-Link Heating Equations. Considering the thermal regime to be symmetrical relative to the half of the fuse-link, the heating is determined by solving the equation (2). With the limiting conditions:

$$\frac{d\tau(x)}{dx}\bigg|_{x=0} = 0\ ;\ \tau(x)\Big|_{x=x_1} = \tau_1 \tag{9}$$

the following solution is obtained:

$$\tau(x) = \tau_1 \frac{chax}{chax_1} + \frac{\rho_a A J^2}{Kl - \alpha\rho_0 A J^2}\left[1 - \frac{chax}{chax_1}\right] \tag{10}$$

while with limiting conditions:

$$\frac{d\tau(x)}{dx}\bigg|_{x=0} = 0\ ;\ -\lambda\frac{\partial \tau(x)}{\partial x}\bigg|_{x=x_1} = \eta\,\tau(x)\Big|_{x=x_1} \tag{11}$$

The following solution is obtained:

$$\tau(x) = \frac{\rho_a A J^2}{Kl - \rho_0 \alpha A J^2}\left[1 - \frac{gchax}{ashax_1 + gchax_1}\right] \tag{12}$$

where:

$$g = \frac{\eta}{\lambda}\ ;\ a = \sqrt{\frac{Kl}{\lambda A} - \frac{\rho_0}{\lambda}\alpha J^2} \tag{13}$$

The relations (10) and (12) are valid in the case of the constant section constrictions as well as for example in Fig. 1.a. However, because of the heat transfer by means of convection, as compared to conduction, in the case of constrictions, might be neglected (k=0), the heatings are obtained by solving equation (3). In the case of limiting conditions of the type (9) the expression of heating is:

$$\tau(x) = \tau_1 \frac{\cos a'x}{\cos a'x_1} + \frac{\rho_0}{\alpha\rho_a}\left[\frac{\cos a'x}{\cos a'x_1} - 1\right]\ ;\ a' = J\sqrt{\frac{\alpha\rho_a}{\lambda}} \tag{14}$$

and in the case of the limiting conditions (11) the following result is obtained:

$$\tau(x) = \frac{\rho_a}{\alpha\rho_0}\left[\frac{g\cos a'x}{g\cos a'x_1 - a'\sin a'x_1} - 1\right] \tag{15}$$

The Heating of Variable Section Constrictions Equations. The differential equation of heating the constriction Fig.1.b, presented in (4), neglecting the heat transfer by means of conduction (k=0), might be also written as follows:

$$(1+xc)^2\frac{d^2\tau}{dx^2} + d\,\tau(x) = -e \tag{16}$$

where:

$$c = \left(\frac{b}{2} - a\right)\frac{1}{ax_1}\ ;\ d = \frac{\alpha\rho_0 I^2}{4\lambda g^2 a^2}\ ;\ e = \frac{\rho_a I^2}{4\lambda g^2 a^2} \tag{17}$$

The solution of equation (16) with the limiting conditions of the type (9) is:

$$\tau(x) = \left(\tau_1 + \frac{\rho_a}{\alpha\rho_0}\right)\frac{r_2 e^{r_1 \ln(1+xc)} + r_1 e^{r_2 \ln(1+xc)}}{r_2 e^{r_1 \ln(1+x_1 c)} + r_1 e^{r_2 \ln(1+x_1 c)}} - \frac{\rho_a}{\alpha\rho_0} \tag{18}$$

$$r_{1,2} = 0{,}5 \pm \sqrt{0{,}25 - \frac{\alpha\rho_0 x_1^2 I^2}{4\lambda g^2\left[\frac{b}{2} - a\right]^2}}$$

THE FUSE-LINKS HEATING CALCULATION. On calculating the heat under the forms established in the previous chapter, it is very important to know the values of the heat transfer coefficients, k and η, for the studied types of fuses. These coefficients being, besides their physical significance when we refer to the manner in which they have been introduced, also operands, we consider it to be necessary that their values should be determined for each type of fuses. In work [3] the values of the heat transfer coefficients, for some types of fuses are presented. Other values are determined from later research-works and calculations.

Constant Section Constrictions Heating Calculation. The one constriction fuse with a constant section has been designed for the case of a ultrarapid 30V, 160A fuse used for the protection of the installation from the top of the railway carriages. The form of the fuse-link is the one shown in Fig. 1 a having the following dimensions: b=1,2 cm; x_1=0.025 cm; b'=0.004 cm; g=0.015 cm. The overtemperature τ_1 of the constriction has been determined by the relation (15) for the abscissa $x=x_1$ (the end of the constriction) and $\tau_{max.}$ respectively x=0 for the centre of the constriction. The value of a' has been calculated according to the relation (14) and is 5,16 cm, and η as determined from experiments and calculations has been 4,6w/cm^2 $^\circ$C. With the values indicated above it has been obtained from the calculations τ_1 =430°C and τ_{max}= 436°C. As can be seen, between the centre of the constriction and its edge, there is a small temperature difference ($6^{\circ}C$).

<u>Variable Section Constrictions Heating Calculation.</u> Because of the fact that the differential equations (4) and (6) are difficult to solve analitically, even approximately, it has been resorted to the electronic computer. The computing program has been conceived for the following sizes of the fuses: $\lambda = 3{:}93 W/cm^{\circ}C$; $\alpha = 4.39\ 10^{-3}\ 1/^{\circ}C$; $\rho_o = 1.61\ 10^{-6}\ \Omega\ cm$; $\theta_a = 20^{\circ}C$; $I=50A$; $a=0.75$ cm; ($a'=0.145$ cm); $x_1=0.175$ cm ($x_1=0.105$ cm); $\theta_{a1}=80^{\circ}C$ ($\theta'_{a1}=200^{\circ}C$); $\gamma = 0.175$ cm ($\gamma' = 0.105$ cm); $K=0.025 W/^{\circ}C\ cm^2$ and $K=0.025\ 10^{-2} W/^{\circ}C\ cm^2$. As can be observed, some sizes have two values, values which correspond to two constrictions made along the fuse.

The result of the calculation for the two constructional variants and for two forms of constrictions (Fig.1b and c), are presented in Fig.2. It is observed that the value of the heat transfer coefficient K, has practically no influence upon the heating value. These results justify the hypothesis of neglecting the convection heat transfer (K=0). It is also observed that in such cases the difference of temperatures between the centre and the edge of the constrictions is small, of some degrees order.

<u>Constrictions Fuse-Link Heating Calculation.</u> The majority of the electric fuses fuse-links are made of strips along which are made constrictions, in order to obtain a limiting effect of the current and a specific type of time-current fuse characteristic. Exact calculation expressions for this type of fuse-link have not been deduced yet. The independent constrictions have been considered, in order to calculate the overtemperatures, by determining the heating for each constriction, separately. The heating of the fuse-link, considered without constrictions has also been determined. The parameters of the 20A ultrarapid fuse-link are the following $J=2.42\ 10^{4} A/cm^2$; $x_1=2{,}4$ cm; $\eta=18 W/^{\circ}C\ cm^2$; $K=0.014 W/^{\circ}C\ cm^2$; $J_{n1}=7.27\ 10^{4} A/cm^2$; $\eta_{n1}=1.15 W/^{\circ}C\ cm^2$; $l_{n1}=0.061$ cm; $A_{n1}= 1.375\ 10^{-4}\ cm^2$; $x_{n1}=0{,}025$ cm $J_{n2}=6.06\ 10^{4} A/cm^2$; $A_{n2} = 1.65\ 10^{-4}\ cm^2$; $l_{n2}=0.071$ cm; $\eta_{n2}=1 w/^{\circ}C\ cm^2$; $x_{n2}=0{,}025$ cm $J_{n3}=5.19\ 10^{4} A/cm^2$; $\eta_{n3}=0.9 W/^{\circ}C\ cm^2$; $l_{n3}=0.081$ cm; $A_{n3} = 1.925\ 10^{-4} cm^2$; $x_{n3}=0{,}025$ cm The heating of the fuse-link considered homogenous, has been determined by the formula (12) and it is represented in Fig.3, while the constrictions heating has been determined by the formula (15). The results of the calculation for the extremity and the centre of the constriction are the following: $\tau_{11}=556^{\circ}C$; $\tau_{max\cdot 1}=564^{\circ}C$; $\tau_{21}=345^{\circ}C$; $\tau_{max\cdot 2}=350^{\circ}C$; $\tau_{31}=225^{\circ}C$; $\tau_{max\cdot 3}=228^{\circ}C$. The graph of the fuse-link heating, taking into account the overtemperature of each constriction is represented in Fig.3. It is possible to observe in this case too, that the difference of the temperature between the extremity and the centre of the constriction is of the order of degrees. It is worthwhile to mention that such curves for constrictions fuse-links are presented in specialized literature [5] [6] with no details concerning

their calculation.

CONCLUSIONS. In order to determine the heating of the electric fuses, fuse-links precise analytic computation relations may be obtained, which are more or less complicated. The main problem is to know the heat transfer coefficients, for the differential equations might be solved by means of the electronic computers (if it is not possible to use the analytical method).

From the calculation of the constrictions heating, results that the temperature difference between the centre and the edge of the constrictions is of the Celsius degrees order. It is also observed that in steady state the overtemperature of the ultrarapid fuses fuse-links is great in the centre of the fuse-link (ex:564°C), that is, of the order of some hundreds of degrees. These heatings correspond to some current densities of the order of some thousands of amperes pro mm^2.

Although the difference of temperature along the constrictions is small, its existence determines a very intensive thermal regime of the fuse. For example the maximum supratemperature of the fuse-link at the same current, without constrictions, is under 300°C, and with constrictions it is of 564°C

BIBLIOGRAPHY

[1] Barbu, I.: Contributii privind unele fenomene electrotermice din segmente nefuzibile si modelarea lor. Rezumatul tezei de doctorat. I.P.Timisoara, Timisoara 1971.

[2] Avramescu, A.: Incalzirea adiabatica a conductorilor din cupru, aluminiu si argint. In: Studii si Cercetari de Energetica, tomul V, 1955, nr.3-4.

[3] Barbu, I.: Consideration sur la transmission de la chaleur dans les coup-circuits unipolaires à filet. In: Lucrarile ICPE nr 21, 1969, p.83-91.

[4] Barbu, I.: Consideratiuni asupra fenomenelor stationare electrice si termice la functionarea sigurantelor fuzibile. In: Electrotehnica 17, nr 4, aprilie 1969, p.137=143.

[5] Lipski, T.: Bezpieczniki niskonapieciowe. Wydawnictwa Naukowo-Technicze Warszawa.

[6] Henselmeyer, G. si Walter, R.; Neue Diazed - Sicherungen. In: Siemensz 33 nr 6, iunie 1959, p.417-424.

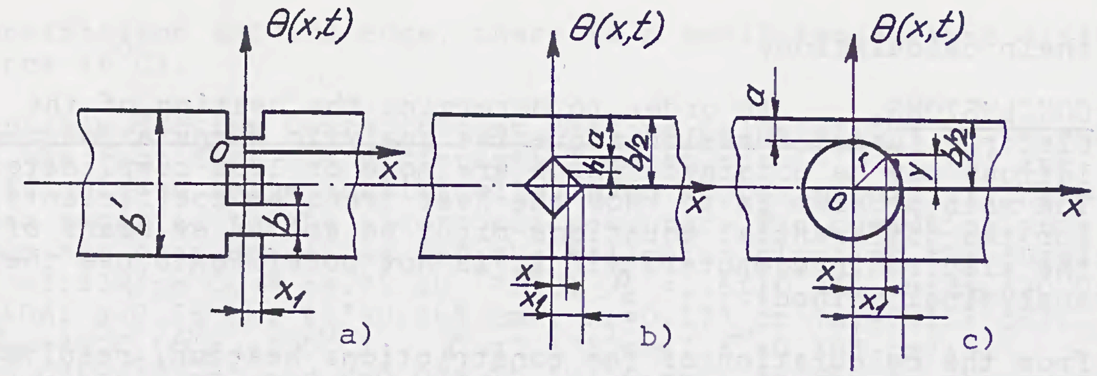

Fig.1 Constrictions types made along the fuse-links

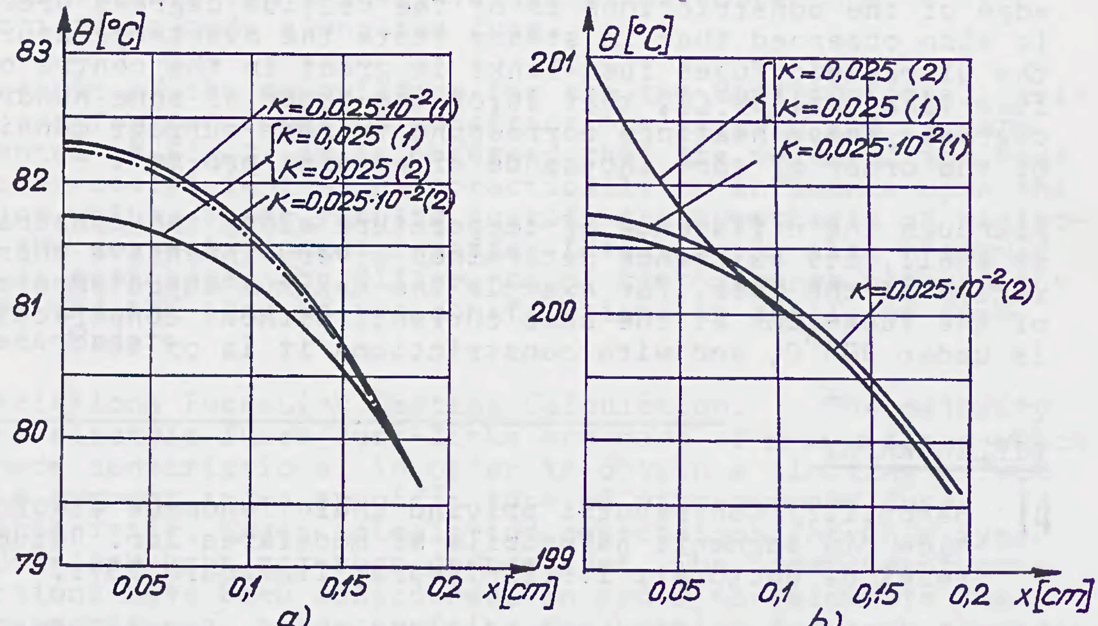

Fig.2 Variable section constrictions heating
a) temperature $\theta_{al} = 80^{\circ}C$; b) temperature $\theta_{al} = 200^{\circ}C$
1: rhombic-shape holes
2: annular-shape holes

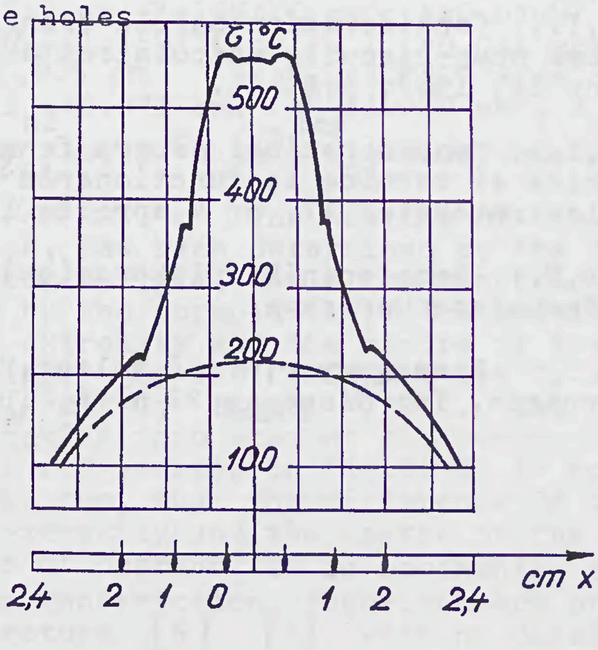

Fig.3 Constrictions fuse-links heating