A DECOUPLED METHOD FOR PREDICTING TIME-CURRENT CHARACTERISTICS OF HRC FUSES

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PRINCIPAL SYMBOLS

C	specific heat
ρ	density
K	thermal conductivity
$\mathcal{K}$	thermal diffusivity
σ	electrical conductivity
α	temperature coefficient
T	temperature
J	current density
M	modal parameter for finite-difference solutions
t	time
v	volume of sub-volume of element
As	sub-volume surface area
M.F.C.	minimum fusing current
I	r.m.s. symmetrical prospective current
Δx, Δy, Δz	spatial separation between nodes
Δt	time step

Suffixes and superscripts are used as follows:

e	pertaining to the fuse-element
f	pertaining to the filler
A	at ambient temperature
m	at melting temperature
i, j, k	spatial identifiers
n	time identifier. Thus $Ti,^{n}j,k$ is the temperature at a point whose coordinates are $i\Delta x$, $j\Delta y$, and $k\Delta z$ at a time $n\Delta t$

INTRODUCTION Calculation of the complete time-current characteristics of practical fuses must be done using numerical methods (1), (2). These fuses usually contain elements with multiple constrictions in the form of notches. Finite difference methods have been found most suitable.

For each value of prospective current the temperature distribution within

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the fuselink is computed as a function of time until melting occurs at some point on the fuse element. In the regime of the time-current characteristic, heat loss from element to filler, and along the element to the end connections is significant in determining the melting time.

However, a full 3-dimensional finite difference model of a practical fuselink requires a vast number of nodes, since the node spacing is fundamentally determined by the need to accurately represent the element thickness and the geometry of the reduced element section. This means that colossal computer running times are necessary to obtain a solution.

Excessive computer storage and running times may be overcome without adverse effect by using a decoupled numerical solution. In this method the transient temperature distribution along the element is computed by conventional methods, but at each step a separate calculation is made of heat lost to the filler from the surface of the element. Axial and lateral heat transfer within the filler, and the effect of the outer cartridge are neglected. These assumptions become invalid for very long melting times (for currents approaching the M.F.C.). In this region a quite different, quasi-steady-state representation is necessary.

The decoupled method has been found to give predictions of time-current characteristics in good agreement with test results for a wide variety of fuselinks.

FUSE REPRESENTATION The decoupled model is shown in Fig.1. The fuse element is divided into subvolumes as shown, thus permitting the solution of the transient (2-dimensional) temperature distribution within the fuse element by finite-difference methods. Nodes are introduced normal to each subvolume to permit calculation of the temperature distribution in the filler adjacent to the element. The following assumptions are made:

(i) the outer cartridge and environmental effects play no part in the processes.

(ii) heat conduction in the filler occurs normal to the subvolume surfaces, i.e. in the y-direction only.

(iii) the current distribution within the element remains unchanged throughout the transient (1), (4).

(iv) K_e and K_f do not vary with temperature.

(v) the fuse end-caps are represented as fixed-temperature boundaries.

(vi) the thermal capacity of each subvolume of the element is lumped at a single node. This is possible because the element thickness is very small compared with Δy.

The conventional time-current characteristic (3) covers the range $0.01s < tm < \infty$. The decoupled model is valid also over this range, except for very long melting times, where assumption (i) becomes invalid. Determination of the M.F.C. must be done using a steady-state representation, in which environmental effects are very important.

SOLUTION METHOD The Joulean heat generated within the element subvolume is determined by computing the current density distribution by numerical solution of the electric field problem[4]. The heat lost to the filler surrounding each subvolume is given by Fourier's Law[5] as:

$$q_f = - K_f \left| \frac{\partial T}{\partial y} \right._{y=o} \cdot A_s \qquad \ldots\ldots (1)$$

The net heat generated within the subvolumes may then be taken as the actual heat generation less the heat lost by conduction to the filler. This is

$$q' = \frac{Ji,j^2}{\sigma_e} + K_f \left| \frac{\partial T}{\partial y} \right._{y=o} \frac{As}{v} \qquad \ldots\ldots (2)$$

(Watts/unit vol.)

This approach is valid because the element thickness is very small compared with the element width and length.

The 2-dimensional temperature distribution within the element is then governed by (5)

$$K_e \frac{\partial^2 T}{\partial x^2} + K_e \frac{\partial^2 T}{\partial z^2} + q' = \rho_e C_e \frac{\partial T}{\partial t} \qquad \ldots\ldots (3)$$

Since heat flows in the filler are assumed normal to each subvolume surface, they are governed by the one-dimensional equation

$$K_f \frac{\partial^2 T}{\partial y^2} = \rho_f C_f \frac{\partial T}{\partial t} \qquad \ldots\ldots (4)$$

The Crank-Nicholson method [1] is used for the iterative numerical solution of (3) for the element temperature distribution. The time step Δt is determined by the element diffusivity and dimensions. This permits the use of the much faster explicit (non-iterative) method for the solution of (4), since the thermal response of the filler is much slower than that of the element.

This gives, for the element (k = 1)

$$T^{n+1}_{i,j,1} = \frac{Me}{2(1+2M_e)}\left\{ T^{n}_{i,j-1,1} + T^{n}_{i,j+1,1} + T^{n}_{i-1,j,1} + T^{n}_{i+1,j,1} - 4T^{n}_{i,j,1} + T^{n+1}_{i,j-1,1} + T^{n+1}_{i,j+1,1} + T^{n+1}_{i-1,j,1} + T^{n+1}_{i+1,j,1} \right\}$$

$$+ \frac{1}{(1+2M_e)}\left\{ T^{n}_{i,j,1} + \frac{J_{ij}^2 \Delta t}{\sigma_A \rho_e C_e}\left[1 + \alpha_e (T^{n}_{i,j,1} - T_A)\right] + \frac{K_f A_s \Delta t}{\rho_e C_e v}\left|\frac{\partial T_{i,j,k}}{\partial y}\right|^{n}_{y=o} \right\} \quad \ldots\ldots (5)$$

and for the filler (k > 1)

$$T^{n+1}_{i,j,k} = T_{i,j,k} + M_f\left\{ T^{n}_{i,j,k-1} + T^{n}_{i,j,k+1} - 2T^{n}_{i,j,k} \right\} \quad \ldots\ldots (6)$$

where $M_e = \frac{K_e \Delta t}{\Delta x^2}$, $M_f = \frac{K_f \Delta t}{\Delta y^2}$ $(\Delta z = \Delta x)$

The surface gradient is obtained by numerical differentiation, (6) using

$$\left|\frac{\partial T_{i,j,k}}{\partial y}\right|^{n}_{y=o} = -\frac{(11T^{n}_{i,j,1} - 18T^{n}_{i,j,2} + 9T^{n}_{i,j,3} - 2T^{n}_{i,j,4})}{6\Delta y} \quad \ldots\ldots (7)$$

The solution proceeds in successive time-steps as follows. The element temperature distribution is found by iterative solution of (5). This gives the boundary values $T^{n}_{i,j,1}$ which are then used for the solution for the filler temperature distribution using (6). (7) then gives the gradients for use in (5) at the next time step.
The algorithm is illustrated by the flow chart shown in Fig.2.

DISCUSSION The use of the explicit method for the filler minimises storage and running time but certain precautions must be taken to avoid propagation of large errors (1), which may occur when $\frac{K_f \Delta t}{\Delta y^2} > 0.5$. This places a restriction on the lowest value of Δy which can be used. ($\Delta y > \sqrt{2K_f \Delta t}$ for stability). However, too large a value of Δy cannot be used as the truncation error increases in proportion to Δy^2.

For quartz filler the thermal diffusivity is very small (typically 3×10^{-3} cm^2 s^{-1}) and this permits very small values of filler nodal spacing (Δy min $\simeq 0.07$ Δt). Instability is avoided in practice by using M = 0.166 to determine Δy. This value gives minimum truncation error for simple problems (7).

The time step Δt must be determined by successive test runs of the program using progressively smaller values of Δt. The computer program is general in that fuses are simply specified by the input data. The melting time is calculated as a function of several prospective currents which are multiples of the M.F.C. Where the M.F.C. is unknown, suitable prospective currents are determined by scaling short-circuit values obtained from the action integral (8). Individual I: tm solutions are terminated when any element subvolume exceeds melting temperature. The prearcing time is then accurately determined by linear interpolation between the latest and the previous maximum temperature values.

RESULTS A range of single notched silver strip elements centrally positioned in quartz-filled ceramic cartridges was used, and the time-current characteristics were determined experimentally and also by using the program. The elements varied in width, length, thickness, shoulder:neck ratio, and body size, but the same filler was used throughout.

The dimensions of each fuse are given in Fig.3, and the time-current characteristics are shown in Fig.4.

Fig.4 shows that the decoupled method gives very good agreement with the experimental curves, up to 2s for notched elements with large shoulder:neck ratios (typically 10:1) and up to 10s for elements with smaller shoulder:neck ratios (of the order of 5:1).

Typical computer running times and storage requirements are shown below.

The data shown refers to the prediction of the complete characteristic of each fuse using a moderately slow business machine (ICL 1901 A).

Fuse type	Running time (Δt = 0.04s)	Storage (words)
1	5h	< 10k
2	8h	< 10k
3	26h	< 8k

For a modern fast computer the running times will be reduced by a factor of about 200.

PHYSICAL DATA

The electrothermal data which was used is given below.

K_e	4.2	$Wcm^{-1}{}_o\,C^{-1}$
K_f	5.86×10^{-3}	$Wcm^{-1}{}_oC^{-1}$
αe	0.00445	$^oC^{-1}$
C_e	0.232	$W\text{-}s\text{-}g^{-1}{}_oC^{-1}$
ρ_e	10.49	$g\text{-}cm^{-3}$
C_f	1.176	$W\text{-}s\text{-}g^{-1}{}_oC^{-1}$
ρ_f	1.8	$g\text{-}cm^{-3}$
σ_A	6.11×10^6	$S\text{-}cm^{-1}$
T_m	960.8	^oC
T_A	22.0	^oC

CONCLUSIONS The decoupled method gives accurate predictions of time-current characteristics without excessive computer time and storage.

The accuracy of predictions was found to be highly dependent upon the choice of time step used. Experience is necessary to determine the maximum economical value, which does not affect the accuracy of the results.

The accuracy of the computed filler heat losses was difficult to assess, but since axial and lateral heat flow in the filler, the influence of the cartridge, and environmental effects are neglected, it must be expected that the computed losses are only approximate for long melting times. The decoupled model will not give accurate results beyond 10s unless an alternative method of computing the losses to the filler is used, based upon quasi-steady state solutions.

ACKNOWLEDGEMENTS Thanks are due to the Computing Services of Preston and Liverpool Polytechnics.

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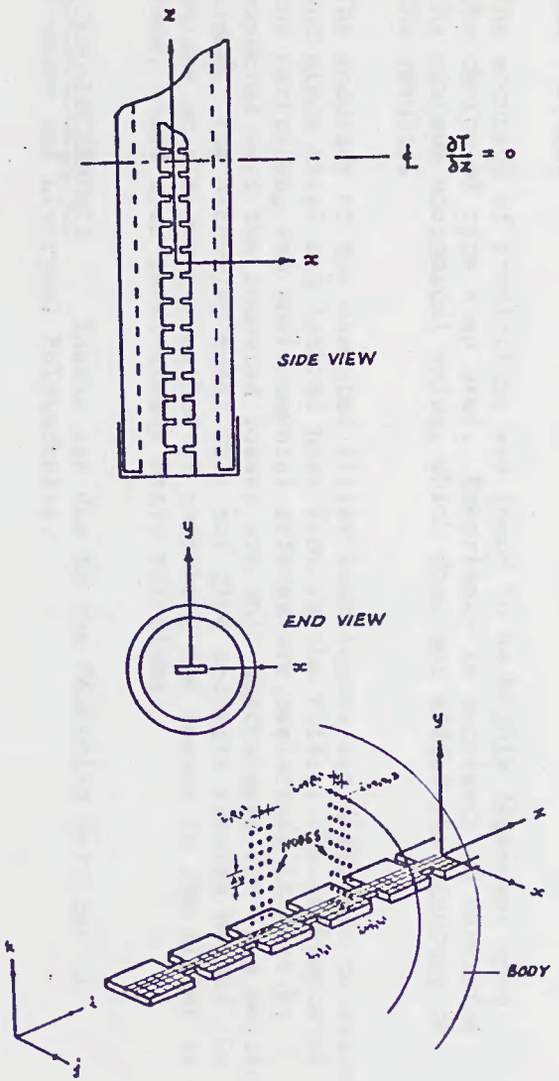
z
$\frac{\partial T}{\partial z} = 0$
x
SIDE VIEW
y
END VIEW
x
y
x
x
BODY
k
i
j

FIG.1

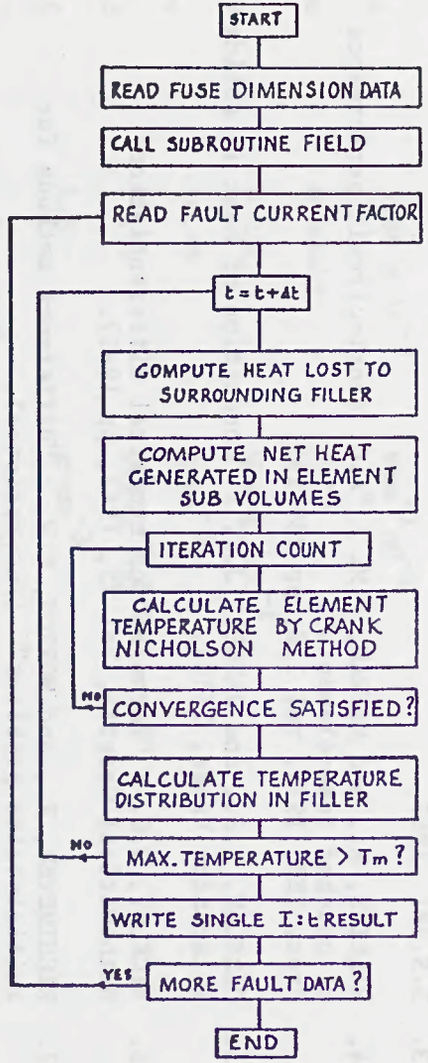
START
READ FUSE DIMENSION DATA
CALL SUBROUTINE FIELD
READ FAULT CURRENT FACTOR
t = t+Δt
COMPUTE HEAT LOST TO SURROUNDING FILLER
COMPUTE NET HEAT GENERATED IN ELEMENT SUB VOLUMES
ITERATION COUNT
CALCULATE ELEMENT TEMPERATURE BY CRANK NICHOLSON METHOD
NO
CONVERGENCE SATISFIED?
CALCULATE TEMPERATURE DISTRIBUTION IN FILLER
NO
MAX. TEMPERATURE > Tm ?
WRITE SINGLE I:t RESULT
YES
MORE FAULT DATA ?
END

FIG. 2

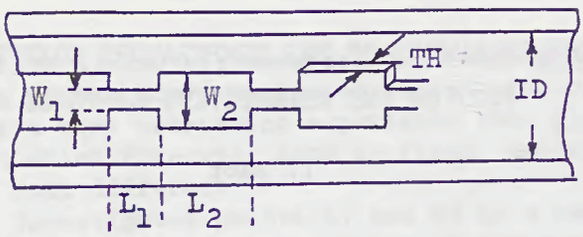

FIG.3

Element type	Number of notches per element	Fuse dimensions (cm)					
		ID	L_1	W_1	L_2	W_2	TH
1	23	1.27	0.161	.07455	0.485	0.1527	.00508
2a	23	1.27	0.066	.0724	0.578	0.3175	.00762
2b	23	1.27	0.066	.0724	0.578	0.3175	.01016
3a	5	0.94	0.0786	.0766	0.714	0.812	.00508
3b	5	0.94	0.0786	.0766	0.714	0.812	.01524
3c	5	0.94	0.0786	.0766	0.714	0.812	.02286

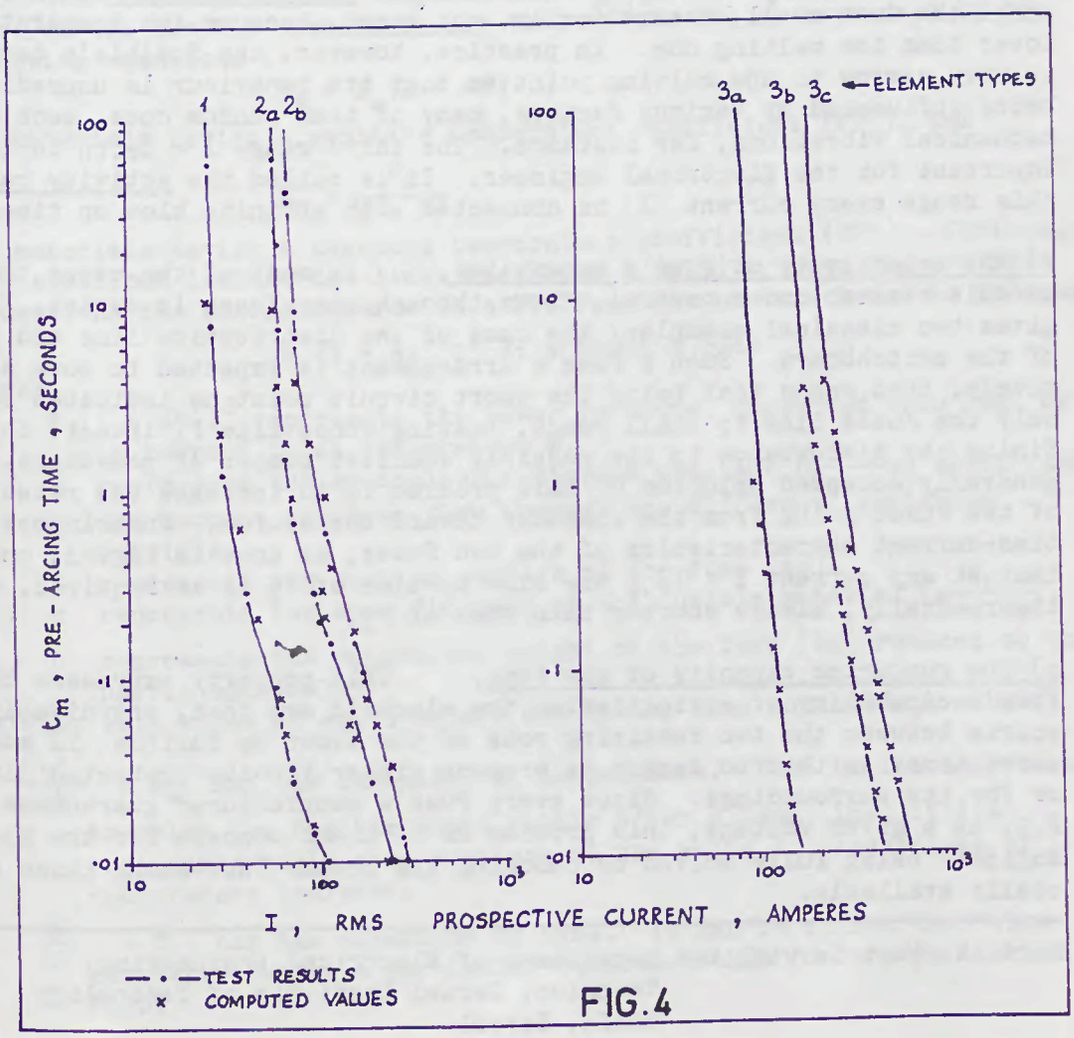

FIG.4