Mathematical Modelling of Thermal Processes in Vacuum Fuses

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Summary

The vacuum fuses, in contrast to conventional quartz filled fuses, have an outstanding certain operation in the range of low currents and excellent arc quenching capabilities [1]. In the paper, the steady state nonlinear equation for heat transfer in the fuse is solved, considering the radiation losses and the conductive heat transfer to the butt electrodes. This makes possible to evaluate the influence of the length and the cross-section of the fuse wire on the minimal operating current. In the case of negligible radiation losses the time-current characteristics are determined by solving the linear heat transfer equation using the Laplace transform and the numerical calculation of Mellin- Fourier integral. The results are compared with experimental data.

1. Introduction

Considering only the radiation losses and controlled current, the heat transfer equation for thin cylindrical fuse wire is

$$\frac{\partial T}{\partial t}=\frac{\lambda}{c\gamma}\frac{\partial^2 T}{\partial x^2}-\frac{2\,c_1\,\varepsilon\,(T^4-T_0^4)}{c\gamma\, r}+\frac{p_0(1+\alpha\theta+\beta\theta^2)}{c\gamma}\quad;\qquad c_1=5.67\cdot 10^{-8}\qquad(1)$$

where ε is the wire radiation constant, c - the wire specific heat, γ - the wire density, r - the wire section radius, $T = 273 + \theta$ is the absolute temperature of the wire at distance x from center, T_0 - the ambient temperature, $p_0 = \rho_0 j^2$ - the specific Joule loss at 0 °C .

The thermal conductivity is decreasing and specific heat is increasing with the temperature and near the melting temperature θ_n they are

$$\lambda=\lambda_0(1-\alpha_\lambda\theta_n)\,;\qquad c\gamma=(c\gamma)_0(1+\alpha_c\theta_n)\qquad(2)$$

where the values at 0 °C are indexed with 0 and given in appendix.

We will choose the linear temperature variation of the resistivity, which gives a good approximation near melting temperature, considering

$$\rho=\alpha_R\rho_0 T\quad;\qquad \alpha_R=\frac{1+\alpha\,\theta_n+\beta\,\theta_n^2}{T_n}\qquad(3)$$

In this case the heat transfer equation becomes

$$\frac{\partial T}{\partial t}=\frac{\lambda}{c\gamma}\frac{\partial^2 T}{\partial x^2}-\frac{2\,c_1\,\varepsilon\,(T^4-T_0^4)}{c\gamma\, r}+\frac{\alpha_R\,p_0\,T}{c\gamma}\qquad(4)$$

In adiabatic case $\lambda = \varepsilon = 0$ and the constant K is

$$K = \int_0^t j^2\, dt = \frac{(c\gamma)_0}{\alpha_R\, \rho_0}[(1-273\alpha_c)\ln\frac{T_n}{T_0} + \alpha_c(T_n - T_0)] \qquad (5)$$

More accurate the values of K can be determined, considering the given in [3] parabolic variation with the temperature of the parameters c, γ and ρ. For T_0 = 293 K they are: $6.41\cdot 10^{16}\ A^2 s/m^4$ for silver and $8.83\cdot 10^{16}$ for copper, that rather agree with the given in [4] values (5.91 and respectively 8.63) then from [5] (7.02 and 9.33).

For short pre-arcing times (near adiabatic regime) better approximations are obtained for smaller, equivalent value of α_R , for which the relation (5) gives the true values of K. Thus, for Ag and Cu α_R must be considered:

	α_R for short pre-arcing time	α_R for long pre-arcing time
Silver	$40.1\cdot 10^{-4}$	$44.0\cdot 10^{-4}$
Copper	$38.6\cdot 10^{-4}$	$45.9\cdot 10^{-4}$

2. Negligible Radiation Losses ($\varepsilon = 0$)

In this case the heat transfer equation becomes linear one and denoting by $T(s, x)$ the Laplace time-transform of the absolute temperature T(t, x) and by $T_0(x)$ the initial absolute temperature of wire, we obtain the equation

$$\frac{\partial^2 T}{\partial x^2} + \nu^2 T = -\frac{c\gamma}{\lambda} T_0(x) \quad ; \qquad \nu = \sqrt{\omega^2 - \frac{c\gamma}{\lambda}s} = \sqrt{\frac{c\gamma}{\lambda}(s_0 - s)} \qquad (6)$$

where $j = \dfrac{I}{\pi r^2}$ is the current density and

$$\omega = \sqrt{\frac{\alpha_R\, \rho_0}{\lambda}} = j\sqrt{\frac{\alpha_R\, \rho_0}{\lambda}}\ ; \qquad s_0 = j^2\frac{\alpha_R\, \rho_0}{c\gamma} \qquad (7)$$

For constant current I and linear dependence of initial temperature $T_0(x) = T_1 + (x - l_1)T_2$, the solution of the equation (6) looks like this

$$T(s, x) = A\cos(\nu x) - \frac{c\gamma}{\lambda\, \nu^2} T_0(x) = A\cos(\nu x) + \frac{T_0(x)}{s - s_0} \qquad (8)$$

Considering the initial temperature uniform (T_2 = 0) and the temperature of the butt electrode (for $x = l_1$) constant and equal to T_1, the Laplace image of the temperature, at distance x from the center of the fuse wire, will be

$$T(s, x) = \frac{T_1}{s - s_0}[1 - \frac{s_0}{s}\frac{\cos(\nu x)}{\cos(\nu l_1)}]; \qquad T(s, 0) = \frac{T_1}{s - s_0}[1 - \frac{s_0}{s\cdot\cos(\nu l_1)}] \qquad (9)$$

The temperature in the center of the fuse wire is the original of this expression that can be obtained by numerical computation of the Mellin-Fourier integral [2]

$$T(t,0)=\frac{1}{2\pi i}\,T_1\int_{\varepsilon-j\infty}^{\varepsilon+j\infty}\frac{e^{st}}{s-s_0}\left[1-\frac{s_0}{s\cdot\cos(\nu l_1)}\right]ds \qquad (10)$$

The time-current density characteristics for silver are given in fig. 1 for different wire length. The adiabatic case corresponds to $L\to\infty$.

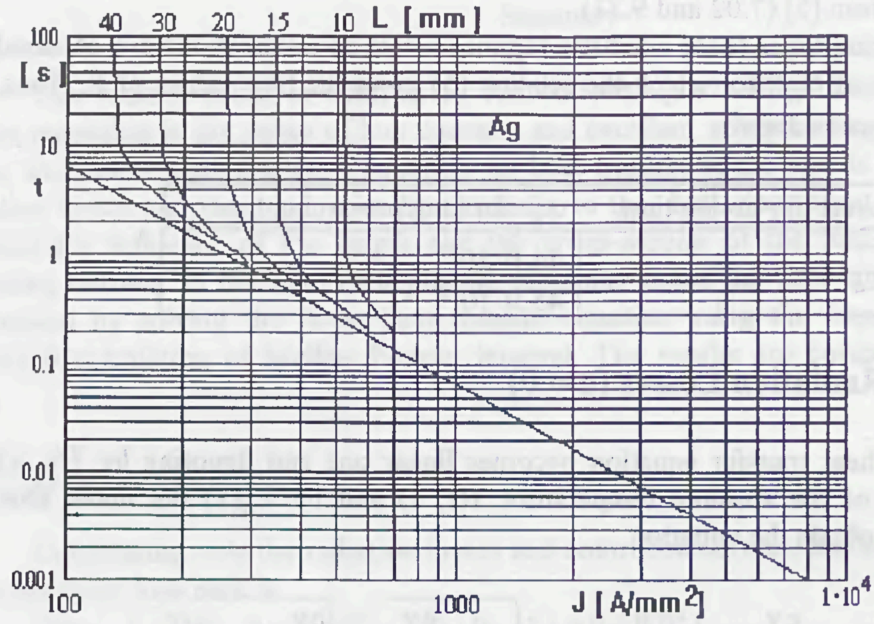

Fig. 1

2.1. The steady-state temperature distribution

The Laplace transform of the temperature in x (9) may be represented like this

$$T(s,x)=T_1\left[\frac{\cos(\nu x)}{s\cdot\cos(\nu l_1)}+\frac{1}{s-s_0}\left(1-\frac{\cos(\nu x)}{\cos(\nu l_1)}\right)\right] \qquad (9')$$

The original of the second term of (9') is [6]

$$f(t)=2\pi\,T_1\,e^{s_0 t}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+0.5)\pi^2}\exp\left[-\frac{(n+0.5)^2\,\pi^2\lambda}{c\gamma\,l_1^2}t\right] \qquad (11)$$

For enough small current densities ($j<j_{cr}$, see below) when

$$(\omega\cdot l_1)<\pi/2 \quad\Leftrightarrow\quad s_0<\frac{\pi^2}{4}\frac{\lambda}{c\gamma\,l_1^2} \quad\Rightarrow\quad \lim_{t\to\infty} f(t)=0 \qquad (12)$$

So, the steady-state temperature may be obtained as the limit of the first term of (9') for $s\to 0$, when $\nu\to\omega$

$$T(x)=\lim_{t\to\infty} s\cdot T(s,x)=T_1\left[\frac{\cos(\omega x)}{s\cdot\cos(\omega l_1)}\right];\quad \omega l_1<\frac{\pi}{2} \qquad (13)$$

If the butt electrode temperature is maintained at θ_1, the rated current of the fuse may be determined from (13) and (7) considering that the melting temperature is reached in the center of the wire: $T(0) = 273 + \theta_n$, i. e. from the condition:

$$T_n = 273 + \theta_n = (273 + \theta_1)\frac{1}{\cos(\omega l_1)} \tag{14}$$

Considering the expression (7) for ω, we obtain for rated current and current density

$$I_0 = C_0 \frac{d^2}{L}; \quad C_0 = \frac{\pi}{2}\sqrt{\frac{\lambda}{\alpha_R \rho_0}} \arccos\frac{T_1}{T_n}; \quad L \cdot j_0 = C_1 = \frac{4}{\pi} C_0 \tag{15}$$

where $d = 2\,r$ and $L = 2\,l_1$ are the diameter and the length of the wire.

For $\omega l_1 = \pi/2$ the steady-state wire temperature becomes infinite. This defines the critical current density j_{cr}, that is the greatest current density at which the Joule loss can be completely evacuated by wire conductivity and consequently the wire steady-state temperature has a finite value:

$$I_{cr} = \frac{\pi}{L}\sqrt{\frac{\lambda}{\alpha_R \rho_0}} \qquad L \cdot j_{cr} = \pi\sqrt{\frac{\lambda}{\alpha_R \rho_0}} = C_{cr} \tag{16}$$

For $\theta_1 = 20°C$ and dimension in mm the constants C_0, C_1 and C_{cr} are the following:

		C_0	C_1	C_{cr}
Silver	A/mm	4018	5116	6037
Copper	A/mm	4535	5548	6629

2.2. Two sections fuse wire

For two cross-sections fuse wire (A_1 and A_2) or in the case of cylindrical butt electrodes in vacuum, denoting by 1 all the parameters of central part and by 2 of the marginal parts, the steady-state solution, satisfying the boundary conditions

$$x = 0 \;\Rightarrow\; T_1(0) = T_m; \quad \frac{dT_1}{dx}\Big|_{x=0} = 0 \tag{17}$$

$$x = l_1 \;\Rightarrow\; T_1(l_1) = T_2(l_1); \quad q = -\lambda_1 A_1 \frac{dT_1}{dx}\Big|_{x=l_1} = -\lambda_2 A_2 \frac{dT_2}{dx}\Big|_{x=l_1} \tag{18}$$

will be:

$$T_1(x) = T_m \cos(\omega_1 x) \tag{19}$$

$$T_2(x) = T_m[\cos(\omega_1 l_1)\cdot\cos(\omega_2(x - l_1)) - f\cdot\sin(\omega_1 l_1)\cdot\sin(\omega_2(x - l_1))] \tag{20}$$

where

$$f = \frac{\lambda_1 \omega_1 A_1}{\lambda_2 \omega_2 A_2} = \sqrt{\frac{\alpha_{R1}\rho_{01}\lambda_{01}}{\alpha_{R2}\rho_{02}\lambda_{02}}}; \quad \omega_i = \sqrt{\frac{\alpha_{Ri}\,\rho_{0i}}{\lambda_i}} = j_i\sqrt{\frac{\alpha_{Ri}\,\rho_{0i}}{\lambda_i}}; \quad i = 1, 2 \tag{21}$$

For $x = l_1 + l_2$ we obtain the butt electrode temperature

$$T_2(l_1 + l_2) = T_m[\cos(\omega_1 l_1)\cdot\cos(\omega_2 l_2) - f\cdot\sin(\omega_1 l_1)\cdot\sin(\omega_2 l_2)] \tag{22}$$

3. Influence of Radiation Losses on Steady-State Temperature Distribution

Considering $\dfrac{\partial T}{\partial t} = 0$ in (4) we obtain the steady-state equation

$$\frac{d^2T}{dx^2} = a\,(T^4 - T_0^4) - \omega^2\,T; \qquad a = \frac{2\,c_1\,\varepsilon}{\lambda\,r} \quad [1/m^2] \tag{23}$$

Multiplying the equation by $\dfrac{dT}{dx}$ and integrating, the following solution, satisfying the initial conditions (17), is obtained

$$\frac{dT}{dx} = -\sqrt{\omega^2(T_m^2 - T^2) + 2\,a\,[T_0^4\,(T_m - T) - 0.2\,(T_m^5 - T^5)]} \tag{24}$$

A second integration give us the steady-state temperature distribution along of central part of fuse wire

$$x(T) = \int_{T/T_m}^{1} \frac{dz}{\sqrt{\omega^2(1 - z^2) + 2\,a\,[\dfrac{T_0^4}{T_m}(1 - z) - 0.2\,T_0^3(1 - z^5)]}} \tag{25}$$

The integral may be calculated with the 32-point Gauss quadrature subroutine DQG-32 in FORTRAN.

For a very long wire, when $L \to \infty$, the wire temperature may be considered constant ($\dfrac{dT}{dx} = 0$) and the equation (23) becomes

$$a\,(T^4 - T_0^4) = \omega^2\,T \tag{26}$$

In this case the limit current is

$$I_\infty = C_\infty d^{3/2}\,; \qquad C_\infty = \frac{\pi}{2}\sqrt{\frac{c_1\,\varepsilon}{\alpha_R\,\rho_0}\,\frac{T_n^4 - T_0^4}{T_n}} \tag{27}$$

For T_0 = 293 °C $\Rightarrow C_\infty = 2.847 \cdot 10^5$ A/m$^{1.5}$ for silver and $1.948 \cdot 10^6$ for copper. Considering the both thermal flux, the limit current of fuse may be approximated by

$$I_n = \sqrt{I_0^2 + I_\infty^2} \tag{28}$$

The dependence of the limit current I_n versus the silver wire length for different wire diameters is given in fig. 2.

4. Comparison with Experimental Data

The limit current for L = 20 mm measured in [1] for silver wire with diameter d = 0.35 mm is 18 A and calculated with (28) is 24.6 A. For d = 0.40 mm the values are respectively 36 A (measured) and 32.2 A (calculated). The differences may be explained by the different thermal contacts between fuse wire and the butt electrodes and by statistical deviations.

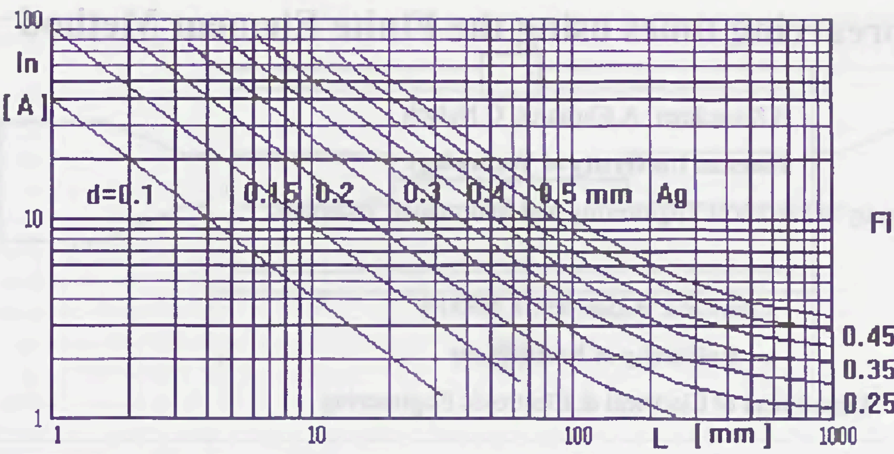

Fig. 2

APPENDIX

Silver and copper wire parameters in SI units for 0 °C [3]

	c	$\gamma \cdot 10^3$	λ	$\alpha_\lambda \cdot 10^4$	$\rho_0 \cdot 10^8$	$\alpha \cdot 10^4$	$\beta \cdot 10^8$	θ_n	ε	$\alpha_c \cdot 10^4$
Ag	235	10.56	418	4.46	1.47	40.3	60	961	0.02	1.54
Cu	386	8.89	388	1.80	1.62	43.3	45.3	1083	0.81	1.20

The specific heat for silver is: $c = c_0(1+2.13\cdot 10^{-4}\theta)$. The specific heat for copper up to 500 °C is $c = c_0(1+1.7\cdot 10^{-4}\theta + 60\cdot 10^{8}\theta^2)$
and $c = c_{500}[1+1.8\cdot 10^{-4}(\theta-500)]$ for $\theta \in (500, 1000)$.
The coefficients of linear expansion are:

$$\alpha_1 = 19.5\cdot 10^{-6} + 1.45\cdot 10^{-9}\theta + 2.25\cdot 10^{-12}\theta^2 \quad \text{for Ag}$$

$$\alpha_1 = 16.7\cdot 10^{-6} + 3.8\cdot 10^{-9}\theta + 1.5\cdot 10^{-12}\theta^2 \quad \text{for Cu.}$$

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