SOME PROBLEMS IN THE MODELLING OF MINIATURE FUSES

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Abstract

Realistic modelling of the thermal behaviour of miniature fuses requires primarily the estimation of heat losses from the fuse element, and the way in which these losses are affected by the element dimensions, fuse body size, type of test clip, and the nature of the filler if used. In the paper the heat loss mechanisms are discussed and some experimental correlations for the heat-loss coefficients are given. Solution methods for the resulting non-linear set of equations are then described. Arcing phenomena for fuses with and without filler are then reviewed and a general approach to the modelling of arcing is presented. Deficiencies in existing knowledge are highlighted as areas for possible future research.

LIST OF SYMBOLS

- m = density of element material
- c = specific heat of element material
- S = element cross-sectional area
- T = $T(x,t)$ = temperature rise of fuse element
- I = element current
- ρ_0 = specific resistivity of element material
- α = temperature coefficient of resistivity
- K = thermal conductivity of element
- d = diameter of wire element
- h = wire surface heat-loss coefficient
- x = axial position along element
- t = time
- N = number of element sections
- $[Q]$ = (N+1) x (N+1) matrix for steady-state solutions
- $[R]$ = (N+1) x 1 matrix for steady-state solutions
- $[T]$ = (N+1) x 1 matrix of unknown temperature rises
- C = coefficient in wire heat-loss correlation
- C_b = coefficient in body heat-loss correlation
- C_e = coefficient in end thermal resistance correlation
- L = fuse length
- D_1 = inner diameter of body
- D_2 = outer diameter of body
- T_b = outer body temperature
- T_e = endcap temperature rise
- f = fraction of element touching body wall
- θ = arctan (D1/L)
- γ = coefficient of expansion of element
- T_{av} = average element temperature rise
- g_f = filler thermal resistance per unit length
- g_b = body thermal resistance per unit length
- g_{ext} = external thermal resistance per unit length
- k_f = filler thermal conductivity
- k_b = body thermal conductivity
- R = element resistance per unit length
- R' = spiral element resistance per unit axial distance
- ϕ = characteristic angle of helix
- $[A]$ = (N+1) x (N+1) matrix for transient solution
- $[B]$ = (N+1) x 1 matrix for transient solution
- T_{ms} = average temperature rise of M-blob
- R_{ms} = effective thermal resistance of M-blob
- C_{ms} = effective thermal capacitance of M-blob
- t_d = disruption time, t_r = voltage rise time
- K_d, K_r = coefficients associated with t_d and t_r
- j = instantaneous current density

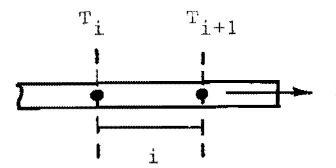

Fig.1 Division of wire element into finite sections.

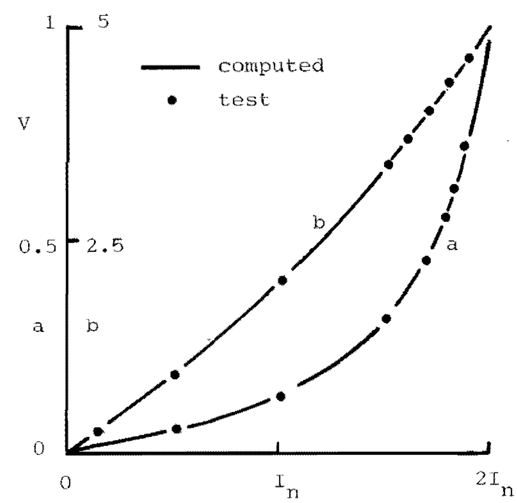

Fig.2 Comparison of test results and computed results with h independent of temperature.

(i) 1A fuse, silver element

(ii) 0.1A fuse, "nickel-silver" element

INTRODUCTION

Computer modelling of fuse behaviour and the simulation of fuse testing plays an increasingly important part in the fuse design process. While there have been several accounts of modelling procedures for sand-filled power fuses, there is very little information available on similar procedures for miniature fuses.

Many miniature fuse designs use wire elements in air, the elements being cooled by natural convection, which is probably the most difficult heat-loss process to model. Some very simplified formulae, such as "Preece's Law" have been commonly used, but these are not sufficiently accurate for design purposes. In practice the effects of elementgeometry and material, body size, and end assembly need to be represented. In this paper thermal modelling is discussed in some detail, and then some initial ideas on the modelling of arcing behaviour are presented.

The fundamental thermal behaviour of a fine-wire fuse element can be accurately represented by the partial differential equation below, which was originally formulated by Verdet [1].

$$mcS \frac{\partial T}{\partial t} = \frac{I^2 \rho_0 (1 + \alpha T)}{S} + KS \frac{\partial^2 T}{\partial x^2} - \pi dhT \quad (1)$$

This equation is a heat balance at point x on the element in terms of the power per unit length. The first term on the right-hand side is the rate of internal heat generation due to the flow of current. The second term on the r.h.s. represents the axial heat loss by thermal conduction along the wire, while the third term is the loss from the wire surface to the surrounding medium. Any imbalance of the terms on the r.h.s. gives rise to a change in internal energy (and temperature), which is given by the term on the l.h.s.

The key to the accurate solution of (1) is knowledge of the surface heat-loss term h, as all the other terms in the equations are known quite precisely.

STEADY-STATE THERMAL SOLUTIONS

By putting $\partial T/\partial t=0$ in (1) we obtain an ordinary differential equation governing the steady thermal state, in which the heat generated is just balanced by the heat losses. This equation can be solved by wholly numerical methods using for example finite differences but there are great difficulties in obtaining satisfactory convergence [2]. If for the moment we assume that h is constant an analytical solution can be found. This solution is given fully in reference [2] which shows that the axial temperature distribution for a finite-length element with constant cross section and properties may be governed by hyperbolic functions (flat-topped - at low currents) or by trigonometric functions (at higher currents). If the current is too high no steady-state solution exists. Equations are also given in [2] for the heat transferred by conduction to the ends of such a finite-length element. This permits an element to be represented by N sections as shown in Fig.1. A model in which the properties of the element vary with x can then be used.

By equating the heat transfer by conduction at the interfaces between sections the following matrix equation is obtained:

$$[Q]\ [T] = [R] \qquad (2)$$

which can be solved for the unknown temperatures [T]. The first and last equations in the set are different from all others in that in these cases the heat transferred by conduction has to be equated to the heat lost to, for example, the test-clips in a temperature-rise test. Rules for assembly of the matrices are given in [2].

In practice the heat-loss coefficients are not constant but are weakly temperature dependent. To allow for this an iterative process is needed. An initial estimate of the heat-loss coefficient is made, after which (2) is solved for the temperatures. From these values better estimates of the heat-loss coefficients may be obtained, (2) is re-solved, and so on, until the process converges, under-relaxation being necessary to ensure convergence.

This iterative analytical method is very fast, as (2) is a tridiagonal matrix equation which permits efficient solutions for the temperatures.

HEAT-LOSS COEFFICIENTS

Given the above model and solution method it was necessary to find an expression for the surface heat-loss coefficient h for a fine wire enclosed in a glass tube without filler. Problems in heat transfer by natural convection are solved by establishing a functional correlation between the Nusselt and Rayleigh numbers, which then permits h to be found for given conditions [3]. Over a given range of Rayleigh number the functional correlation can be modelled as a power-law, with h shown to vary with the physical dimensions of the object losing heat and the temperature difference between the object and its surroundings. There are published data for wires in free air but with a miniature fuse the situation is different, in that the flow of air around the wire is restricted. A 3-dimensional pattern of circulation will be established within the fuse which differs significantly from that which occurs in free air.

Experiments have been conducted to determine the power laws governing (a) the wire surface heat-loss coefficient, (b) the outer body surface heat-loss coefficient, and (c) the thermal resistance of a standard IEC127 test-clip.

(a) wire surface heat-loss coefficient h

First the dependence of h upon wire temperature and diameter was studied by analysing test data on m.f.c.'s , mV drops, and temperature rises for miniature fuses with fixed body size (20mm x 5mm). Fuses with different wire materials, and wire diameters from 0.01mm up to 0.2mm were considered.

It was found that there was no detectable dependence of h upon the wire temperature. Thus for a given fuse geometry, use of a constant value of h gave results predicted by the solution of (1) which fitted the test results very well. Fig.2 shows a comparison of the computed and measured variations of mV drop with current for two different fuses using a constant value of h. If temperature-dependence of h was included using power laws typical of those found for wires in free air [3] the computed results could not be matched to the test data. This result is fortunate as it means that in the iterative solution previously described the alteration of h on the basis on temperature is not necessary, giving faster and more sure convergence.

There was however found to be a weak dependence of h upon the wire diameter, h being found to be approximately inversely proportional to d raised to the power 0.25. This is similar to the dependence of h upon d for wires in free air, and also is in broad agreement with experience over many years with the use of Preece's Law for the minimum fusing current of a wire in air. If (1) is solved in the steady-state for a long wire ($\partial^2 T/\partial x^2=0$) with h independent of d we obtain Preece's Law, i.e. minimum fusing current varies with d raised to the power 1.5. If however h is assumed to vary inversely with d to the power 0.25, the index in Preece's Law falls from 1.5 to 1.375. This trend is evident also in revised versions of Preece's Law which have been published. See for example, reference [4].

Having determined the weak correlation just described, the effect of varying the body size was investigated. This was expected to have a strong influence upon h since the size of the volume of air within the fuse will affect its circulation pattern and hence the value of h. A series of tests was conducted on fuses with varying lengths (up to 32mm) and internal body diameters (3mm to 4.7mm) and in each case a value of h was found by trial and error, which when used in (1) gave the best correlation with the test results. The total series of test results was then analysed to find the indexes in the power-law relating h to the fuse body dimensions. A least-squares best fit gave the following results:

$$h = \frac{C}{d^{0.25}} \left[\frac{20}{L}\right] \cdot \left[\frac{3}{D1}\right]^{0.4} \qquad W / mm^2 / degC$$

where C=3.06 x 10-4 with the dimension in mm.

(b) outer body surface heat-loss coefficient

After (1) has been solved the heat lost radially through the fuse body can be calculated by subtracting the power lost to the element ends (by conduction) from the total generated power. This permits an estimate of the temperature rise of the outer fuse body if the body surface heat-loss coefficient is known. Values of h_b which gave agreement with the test values were found and then a least-squares fit gave the correlation:

$$h_b = \frac{C_b}{D2^{0.584}} \left[T_b\right]^{0.079} \left[\frac{D2}{L}\right]^{0.8} \qquad W / mm^2 /degC$$

where C_b=23.5 x 10-5 with the dimension in mm. This correlation is similar in form to standard formulae for the heat loss from finite horizontal cylinders [3], with a weak dependence of h_b upon temperature.

(c) thermal resistance of test-clip

Heat transferred to the fuse elements ends by thermal conduction along the element is then lost by conduction along the test-clip and by convection and radiation from the surface of the test-clip and the fuse endcap. Again by using similar analytical procedures the thermal resistance viewed from the element ends was found to be governed by the correlation.

$$G_e = \frac{C_e}{(T_e)^{.129}(D2)^{.339}} \qquad deg\ C / W$$

where C_e=294.0. G_e decreases with temperature T_e because of the effect of loss by convection and radiation from the surface of the test clip and the fuse endcaps, while the variation with D2 indicates the relative importance of the losses from the surface of the endcaps.

The three correlations given above represent the major factors which influence the heat loss from an enclosed fuse element in air. There are minor influences also, particularly the effect of wall thickness and endcap length and thickness but the scatter in test data was too large to enable these effects to be isolated.

Nevertheless, using the three correlations above and the iterative analytical solution method, the correlation coefficients relating the computed values of mV drop, body temperature, and endcap temperature to the test values were 98.9%, 92.9% and 97.6% respectively.

EFFECT OF WIRE EXPANSION

When calculating the steady-state thermal performance it is necessary to consider the possibility that owing to thermal expansion the fuse element may defect laterally and touch the inside of the body wall. When this occurs (mostly with longer fuses) the part of the element in contact with the body wall is subject to additional cooling, which typically can lead to an increase of around 15% in the minimum fusing current, together with a localised hotspot on the outer surface of the body near the point of contact. A rough correction for the effect of wire expansion can be made by assuming that the wire deflects in the shape of a parabola [5], and if the expansion, which is proportional to the average wire temperature, exceeds a critical value, the wire will touch the inner wall. If the critical average temperature is exceeded it can easily be shown that the fraction of the wire length in contact with the body is given by:

$$f = 1 - \frac{2}{3} \cdot \frac{\tan^2\theta}{\gamma T_{av}} \qquad (3)$$

If the wire was originally centrally located within the fuse, in which case the fraction in contact will be around the centre of the wire. On the other hand, if the element was originally diagonally wired the element will be pressed up against the body at the ends, and the fraction in contact will be given by:

$$f = 1 - \frac{2}{3} \cdot \frac{\tan^2\theta \ \cos\theta}{(1 - \cos\theta + \gamma T_{av})} \qquad (4)$$

The angle θ is a constant for a particular fuse design. Solution of (3) or (4) with f=0 gives the critical average element temperature for touching.

This model can be incorporated in the iterative solution algorithm as follows. After the element temperatures have been initially computed a check is made to see whether the average temperature is greater than the critical value T_c. If so all elements sections are assumed to be touching and the temperature distribution is recomputed. The fraction f is then reduced after each iteration until the average temperature is just sufficient to cause the assumed value of f. The process is then terminated. For any section of the element in contact with the body the heat-loss coefficient is simply increased by a factor K. Experience indicates that a value of 1.33 typically gives the required correction.

Sometimes after the first iteration it may appear that no steady-state solution exists, the cooling being insufficient for the level of test current applied. If this situation occurs it is necessary to continue the iteration assuming the wire is touching the body, since this may have been the reason for the failure to obtain a solution at the first iteration.

Attempts to model this process with more accuracy met with a difficulty. Sometimes during the computation the wire would expand, touch the body, be cooled, then contract away from contact with the body, setting up a stable oscillation. (This may be possibly occur in some fuses).

However the effect of wire touching the body is a second-order effect and the simple model described can give a reasonable correction to the solution which would otherwise be obtained.

FUSES WITH FILLER

If the fuse is filled the heat loss from the element is by conduction through the filler and body and thence by convection and radiation to the ambient. The solution process described in [2] may be used, which includes a correction for axial heat-flow through the filler and body to the endcaps.

The effective wire surface heat-loss coefficient is then given in terms of the radial thermal resistances, as:

$$h = \frac{1}{\pi d\,(g_f + g_b + g_{ext})}$$

where

$$g_f = \frac{1}{2\pi k_f} \ln \frac{D1}{d} \;;\quad g_b = \frac{1}{2\pi k_b} \ln \frac{D2}{D1} \;;\quad g_{ext} = \frac{1}{\pi D2\, h_b}$$

The appropriate value of h_b is used after each iteration to find the new body temperature and to correct the value of g_{ext} and then h. Otherwise the model is as described in [2].

SPIRALLY-WOUND ELEMENTS

These are commonly used in miniature fuses to achieve anti-surge performance and can be modelled by a very simple variation to the methods described as for plain wire fuses. If the resistance per unit length of the would element is R' per unit length compared with R for the plain wire then

$$\frac{R'}{R} = \frac{1}{\cos\phi} \qquad (5)$$

where ϕ is the characteristic angle of the helix. If the spiral element is regarded as a composite conductor the internal heat generation per unit length and the stored energy are increased by $(1/\cos\phi)$ times compared with a plain straight wire, while the axial thermal conduction is reduced by a factor coso because of the longer axial heat flow path. Using these factors in Verdet's equation (1) it is apparent that the effect of spiral winding can be represented by:

(i) dividing the element thermal capacity by $(\cos\phi)^2$

(ii) multiplying the element cross-section by $\cos\phi$

With these simple alterations the thermal equations for the spiral element became identical with those for the plain element, and the same solutions subroutines can be used. The only extra requirements are that (a) in calculating the heat-loss coefficient for the composite conductor the effective overall diameter of the wound element needs to be used, rather than the wire diameter, (b) a further component must be added to the heat-loss term to account for heat losses from the element to the core of the helix. These losses are only present for transient conditions and are best modelled by representing the core by a lumped thermal reistance-capacitance model.

TRANSIENT THERMAL CALCULATIONS

In this case no analytical solutions of (1) are possible and so numerical methods must be used. The most successful method has been to represent the element by a series of finite sections as shown in Fig.1 and then to develop for these sections a finite-difference equation equivalent to (1) using the Crank-Nicholson technique applied to every term in (1). This results in a tridiagonal matrix equation

$$[A]\ [T] = [B] \qquad (6)$$

which has to be solved at each time step for the transient temperature of the element sections. Some of the elements of the matrices [A] and [B] have to be adjusted at each time-step to allow for temperature-dependence of heat-loss coefficients.

There is little information on the time-dependence of convective loss coefficients but for fuse simulations there is little error in using the steady-state values throughout. This is because the heat-loss terms make very little difference to the thermal response at high currents, where the element heating may be regarded as adiabatic, or for times of the order of 1 sec, where heat losses by conduction to the ends, which is automatically taken into account, is dominant. The main area of difficulty in modelling is for response times of the order of hundreds to thousands of seconds where accurate representation of the temperature variations are currently not possible.

An important practical consideration in automatic computation of thermal transients is the choice of time-step. This must be small enough to give accuracy but large enough to minimise computing time. The following method has been found to give good all-round performance:

(i) Initially compute the melting time neglecting axial heat conduction and using estimates for the heat loss coefficients based upon some convenient temperature rise, say 100°C. This melting time is obtained by a simple analytical solution of (1) with $\partial^2 T/\partial x^2 = 0$.

(ii) Begin the computation with a time step of 1/20th of the "long-wire" melting time, and if melting has not occured after 20 time steps the time step is increased by 50%. This procedure is repeated every 20 time steps.

In the case of filled fuses the above increase of time step is not possible if the transient conduction loss to the filler is computed as this places a limit on the maximum possible time step. However for the simpler model used here based upon the use of an effective value of h to represent the heat loss from the element, the above algorithm has been found satisfactory.

M-EFFECT

Minature fuses often use wire elements with a blob of low melting-point material to achieve M-effect. The thermal mass of this blob is large compared with that of the element to which it is attached, so there is an appreciable thermal lag. Heat is conducted from the element to the M-blob, and the rise in temperature of the M-blob may be modelled as a simple thermal lag, i.e.

$$\frac{dT_{ms}}{d_t} = \frac{1}{R_{ms}C_{ms}} (T_i - T_{ms}) \qquad (7)$$

where T_i is the temperature of the element section to which the M-blob is attached. If Tm_s exceeds the eutectic temperature of the M-blob/element interface, it is assumed that diffusion begins and using the square-root law the eventual breaking time can be estimated [2].

Fuses which use plated-wire to achieve the M-effect are difficult to represent. The only effective way available at present is to treat the wire as a composite with appropriately weighted values for all the thermophysical properties, and a hypothetical M-spot to represent the diffusion which will take place at the interface of the plating with the substrate material.

ARCING PHENOMENA

For breaking-capacity tests on miniature fuses the element melting times will be roughly in the range 0.1ms to 1ms, which gives current densities at melting of 43-13 kA/mm^2 (inductive circuit) or 25-7 kA/mm^2 (resistive circuit). The physical phenomena which occur in this range of current density have been studied by Vermij [6] and Arai [7]. In this range striated disintegration of the element occurs, there being insufficient time for the formation of unduloids.

In order to model the behaviour it is necessary to calculate the fuse voltage at any instant. This is much more difficult than with power fuses, which usually use notched elements in sand, giving arcs with relatively well-defined geometries. For wire elements a large number of small arcs is initially produced and after a very short time these arcs coalesce to form a single arc. This process of coalescence occurs at a crucial phase in the breaking process and more information is needed for satisfactory modelling. The overall breaking time may be divided into four stages:

(a) prearcing phase
The transient temperature response can be obtained using the methods already given. At each time step the fuse voltage is calculated by adding the voltage drops across all the element sections. The effect of element resistance during the prearcing period can very significantly alter the prospective current wave. The prearcing period ends when the element material reaches its melting-point.

(b) disruption phase
In this phase the wire deforms mechanically and eventually breaks. This phase is terminated when the first arc appears. Arai's experiments showed that the disruption time in air was inversely proportional to the current density i.e. $t_d = Kd/_j$ with $K_d = 0.27$ $A\text{-}s/mm^2$. For wires in air the disruption time was longer, and Arai attributed this to the effect of sand grains touching the wire surface and somehow accelerating the disruption process. Vermij also found that disruption times in air were about 50% longer than in sand.

If the disruption process were simply determined by the input energy we might expect t_d to be proportional to $1/j^2$, but if there are mechanical delays in accelerating liquid metal, then for increasing current densities the time required would be longer than that needed simply on the basis of energy input. This implies a dependence of t_d upon j with a power less than 2, as predicted by Arai.

(c) voltage rise phase
This phase has been investigated by Vermij [6] for fuse elements in air and was found to be characterised by a rapid rise in fuse voltage to a value V_0. It is believed that during this phase the voltage builds up in a series of steps as multiple arcs are formed. The peak voltage V_0 at the end of this period can be predicted for wires in sand by the use of Hibner's empirical formula [8]. For wires in air the initial voltage is only about 30% of this value.

The voltage rise time was found to by Vermji to be given by $t_r = K_r/j$ with K_r about 0.2 for wires in air and 0.4 for wires in sand.

These simple rise times can be easily incorporated in computer models see e.g. reference [9], but as the rise times are very short there is little effect upon the results for highly-inductive circuits, in which the circuit current can be taken to constant during the disruption and voltage-rise phases. However many miniature fuse tests are done near to unity power factor, for which a true dynamic representation is needed. In this case the circuit current falls rapidly as the voltage rises thus invalidating Hibner's formula, and (possibly) altering the overall rise time. One method which has been found to give reasonable results is to assume that the fuse resistance builds up during the rise time t_r (rather than the voltage). With this model the 'saturation' effect noted by Baxter [10] can be correctly predicted. (i.e. the fact that the peak voltage reaches a fixed level as the circuit inductance is increased). Nevertheless much more information is needed upon the factors which affect the build-up of fuse resistance, particularly the effect of element material, since miniature fuses use a much wider variety of materials than have been used in the experiments upon which these models have been based.

(d) arcing period
In this period the multiple arcs first coalesce and then a single arc is produced which burns until the current is reduced to zero. For sand-filled fuses an adaptation of the models already used for power fuses can be used, but for wires in air the situation is much more problematical. Some assumptions have to be made about the way in which the initial arc expands to fill the tube. This is another area where much more experimental data is needed. Once the tube is filled we have a wall-stablilised arc with well-defined geometry and which can be modelled much more accurately [9].

CONCLUSIONS

The paper has discussed some of the problems in modelling the behaviour of miniature fuses, and described some solutions which have been found most effective.

General-purpose software for miniature-fuse design has been developed based upon the models described here. In some cases the models used are precise while in others they are based upon 'educated guesswork'. In the latter cases the availability of properly-structured software mean that improvements in modelling can be incorporated as more knowledge is forthcoming as a result of experimental research. It is important to note that assessment of the quality of a certain model is much easier when the software is already available into which the new model may 'plugged' and tested. This leads to the interesting conclusion that the development of software in this area is useful even if all the processes cannot be modelled adequately.

Outstanding areas for miniature-fuse research appear to be: (i) the thermal models for long-duration transients (ii) dynamic investigation of the initial phases of arcing for wires in air, and (iii) further investigation of the melting behaviour of plated wires.

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