CURRENT DISTRIBUTION
IN VARIABLE SECTION FUSES

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Abstract: The thermal and electro-dynamic effects of the current in the fuse depend essentially on the local value of the current density. Using the conformal mappings, the constriction resistance, the current distribution in variable section flat fuses is studied. In particular, the current distribution in arbitrary angle corner and the corresponding resistance is evaluated. Transitory parameters of monophase line with rectangular bar conductors are determined.

Keywords: Current distribution. Transitory parameters. Corner resistance. Constriction resistance.

1. Introduction

The fuses elements for low voltage and large rating currents have in general large cross-section and irregular shapes. This is why the common simplification used for filiform circuits are not more applicable in this case: the current distribution cannot be considered uniform in corners or in regions with sharp cross-section modification and the electromagnetic field in conductor is not established instantaneous when a current step is applied. In the paper some such situations are analyzed and the effects on the element parameters are evaluated.

2. Constriction resistance of bandwidth reduction

The analytical function

$$z(t)=\frac{a}{\pi}\left[\begin{array}{l}\mathrm{Arch}\,\dfrac{2t-x^2-1}{x^2-1}\\ -\dfrac{1}{x}\mathrm{Arch}\,\dfrac{(x^2+1)t-2x^2}{(x^2-1)t}\end{array}\right];\; x=\frac{a}{b} \qquad (1)$$

maps the upper half plane t into the shaded domain from fig. 1 a (fig. 1b) [1].

The "constriction resistance" is defined as a difference between the real resistance of the constricted from a to b band and the sum of the resistances of the two segments, for the case $\overline{\mathrm{PN}}\to\infty$, $\overline{\mathrm{MQ}}\to\infty$:

$$R_s\left(\frac{a}{b}\right)=\frac{1}{\sigma d}\lim_{\substack{r\to 0\\ R\to\infty}}\left[\frac{1}{\pi}\ln\frac{R}{r}-\frac{\overline{\mathrm{PN}}}{a}-\frac{\overline{\mathrm{MQ}}}{b}\right] \qquad (2)$$

where d is the (constant) thickness of the band and σ the material conductivity.

After the computations, similar to given in annex of [4], the following formula was obtained for the constriction resistance of the d thickness band (fig. 2):

$$R_s(x)=\frac{x}{\sigma\pi d}\left[\left(1+\frac{1}{x^2}\right)\ln\frac{x+1}{x-1}+\frac{2}{x}\ln\frac{x^2-1}{4x}\right];\ [\Omega] \qquad (3)$$

The resistance of the shaded band from fig. 1 a) is equal to the sum of the resistances of two segments (with uniform distributed current) plus R_s.

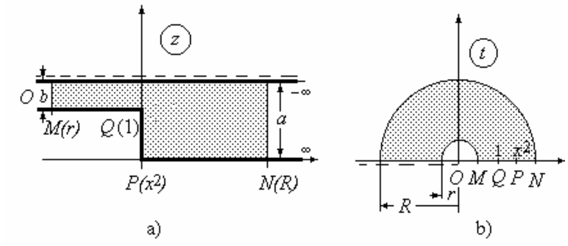

Fig. 1: Domain of the map for "constriction resistance"

The complex potential of the current density field δ in t-plane will be:

$$W=-i\frac{I}{\pi d}\mathrm{Ln}t;\quad t=\rho e^{i\theta}$$
$$\delta=-i\overline{W'}=\frac{I}{\pi\rho d}e^{i\theta} \qquad (4)$$

where I [A] is the current and the complex conjugate of current density in z-plane is

$$\overline{\delta} = i\frac{\mathrm{d}W}{\mathrm{d}z} = i\frac{\mathrm{d}W}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}z} = \frac{I}{a\,d}\sqrt{\frac{t-x^2}{t-1}} \tag{5}$$

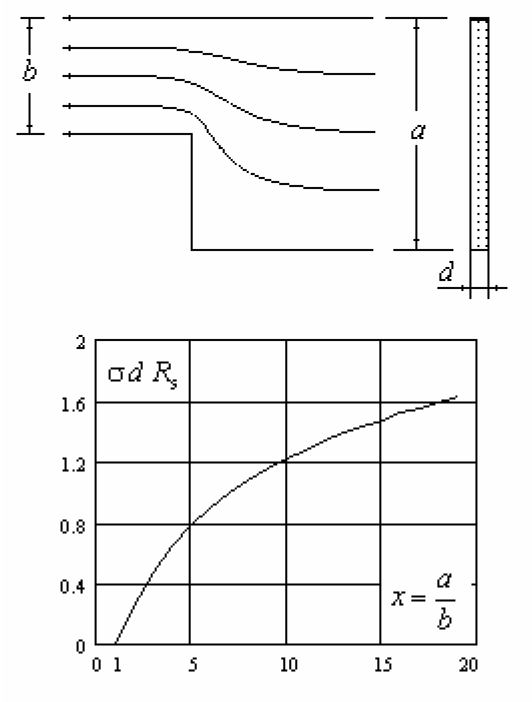

Fig. 2: Constriction resistance of bandwidth reduction

3. “Corner resistance”

A supplementary resistance, which we will call “corner resistance”, must be considered in the case of the slab having the shape shown in fig. 3.

The analytical function

$$z(t) = \frac{b}{\pi}\int_t^1 \left(\frac{1-t}{1+t}\right)^{\beta} \frac{\mathrm{d}t}{t}; \; t = \rho\, e^{i\theta}$$
$$z = x + iy; \quad \beta = 1 - \alpha/\pi \tag{6}$$

maps the shaded domain between the two angles equals to α from the z- plane (Fig. 3) into the upper half-plane t [1].

The corner resistance can be defined as a limit:

$$R_{\mathrm{c}}(\alpha) = \frac{-2}{\sigma\pi d}\lim_{\substack{\rho_0\to 0\\ z_0\to\infty}}\left[\ln\rho_0 + \frac{\pi}{b}z_0\right]$$
$$\frac{\pi}{b}z_0 = \int_{\rho_0}^1 \left(\frac{1-r}{1+r}\right)^{\beta}\frac{\mathrm{d}r}{r} \tag{7}$$

The values of corner resistance are given in fig. 4 and can be approximated with the formula:

$$R_{\mathrm{c}} = \frac{1}{\sigma d}\left(0.882 - 0.324\frac{\alpha}{90°}\right) \;\; [\Omega] \tag{8}$$

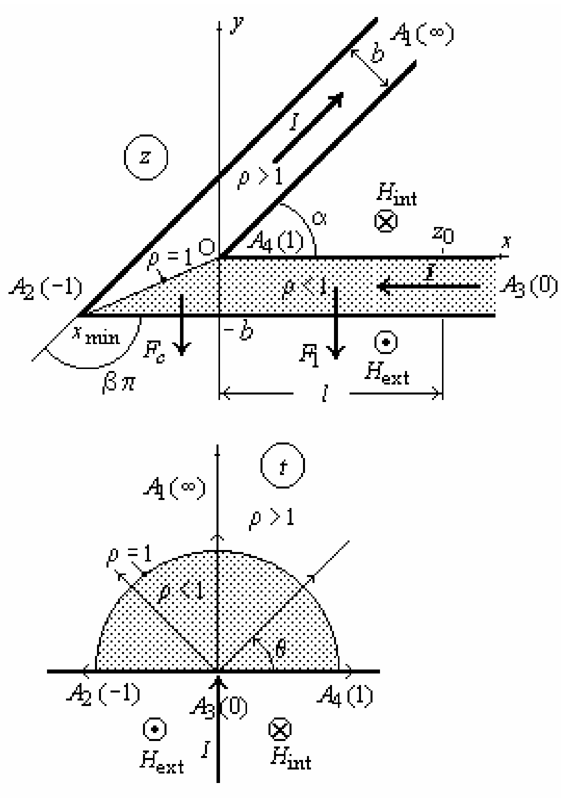

Fig. 3: Domain of the map for "corner resistance"

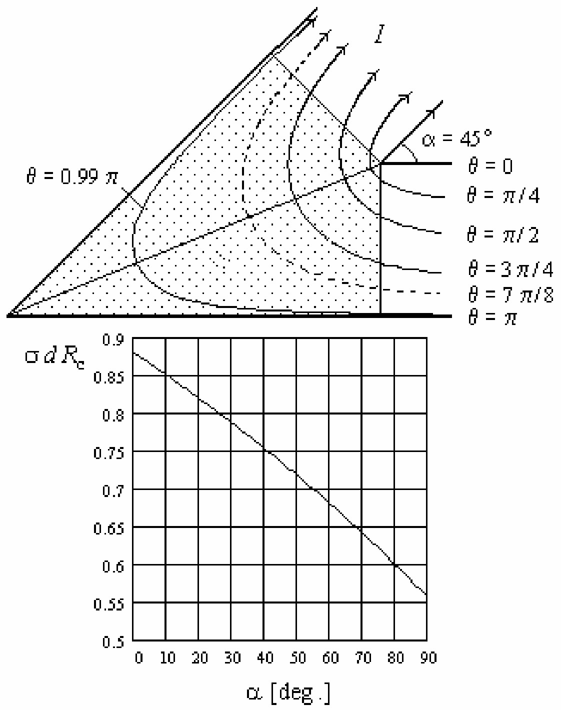

Fig. 4: Corner resistance

The complex conjugate of current density in z-plane (fig. 3 and 4) is:

$$\bar{\delta} = i\frac{\mathrm{d}W}{\mathrm{d}z} = i\frac{\mathrm{d}W}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}z} = -\frac{I}{b}\left(\frac{1+t}{1-t}\right)^{\beta} \tag{9}$$

4. Transitory parameters of rectangular bus bars

4.1. Magnetic field of singular bar

The transitory parameters of two identical very high and close disposed rectangular bars, when the external magnetic field can be neglected, are determined in [3], where a complete smart solution is given for transitory electromagnetic field in such a line. In the paper, the external magnetic field H_B is approximately considered. Assuming the bars enough high to neglect the horizontal component of magnetic field, and consequently the magnetic field constant along the vertical direction, the problem is solved in function of the ratio η between the magnetic fields on the two sides of the bar. The obtained results, for $\eta = 0$ coincide with the given in [3].

4.2. Magnetic field of two rectangular bars

The vertical component of magnetic field of a solitary rectangular bar, with uniform distributed current density δ, issued from the Biot-Savart law, is given in [1] and can be written as follows:

$$H_y(x,y) = \frac{\delta}{2\pi}\begin{bmatrix} (x+a_1)(\theta_2-\theta_3) - \\ (x-a_1)(\theta_1-\theta_4) + \\ (y+b_1)\ln\frac{r_2}{r_1} - \\ (y-b_1)\ln\frac{r_3}{r_4} \end{bmatrix} \tag{10}$$

For a vertical infinite-length current sheet I, a very thin bar, with $a_1 \to 0$, can be considered and the vertical component of the magnetic field (10) becomes:

$$H_y(x,y) = \frac{I}{4\pi b_1}\varphi \tag{11}$$

where φ is the angle at which the sheet cross-section is seen from the point (x, y).

In the case of two parallel rectangular bars we will denote by η the ratio of the magnetic fields H_B / H_A for stabilized direct current and by η_0 the same ratio for the beginning of the process of current vertical infinitely thin sheets.

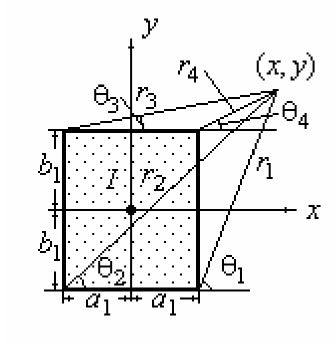

Fig. 5: Rectangular bar

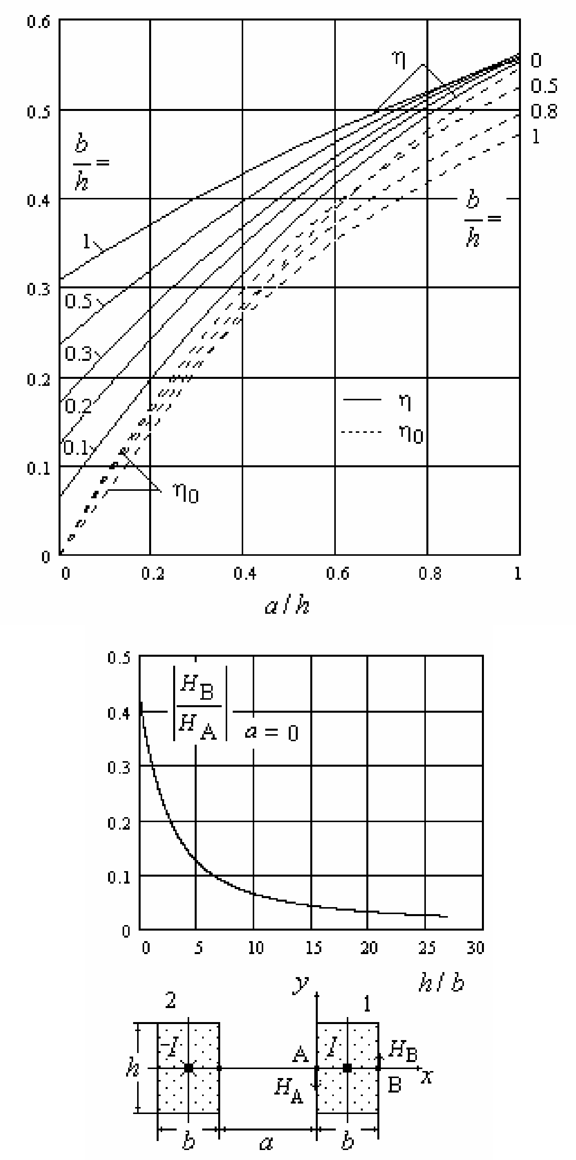

Fig. 6: Ratio of the magnetic field on the external side of the bar to the magnetic field on the internal side, for superficial and uniform distributed currents in two parallel rectangular bars, as function of *a/h* and *b/h*

We will assume a constant current density in each vertical sheet.

Using the function:

$$f(x) = \frac{2x}{h}\arctan\left(\frac{h}{2x}\right) + \ln\sqrt{1+\left(\frac{2x}{h}\right)^2} \quad (12)$$

the two ratio can be written as follows:

$$\eta = \frac{f(a+b)+f(b)-f(a+2b)}{f(a+b)+f(b)-f(a)}$$

$$\eta_0 = \frac{\arctan\frac{h}{b} - \arctan\frac{h}{2a+b}}{\arctan\frac{h}{b} + \arctan\frac{h}{2a+3b}} \quad (13)$$

From the fig. 6 we can observe a small variation of the ratio of the magnetic fields on the two sides from the beginning to the end of current step injection.

4.3. Electromagnetic field in the conductor at current step injection

In the case of current step injection, the Laplace transforms of electric and magnetic field in central part of conductor satisfy the Maxwell equations:

$$\begin{cases} \frac{\mathrm{d}E}{\mathrm{d}x} = p\mu H \\ \frac{\mathrm{d}H}{\mathrm{d}x} = \sigma E \end{cases} \Rightarrow \begin{cases} \frac{\mathrm{d}^2E}{\mathrm{d}x^2} = p\mu\sigma E \\ \frac{\mathrm{d}^2H}{\mathrm{d}x^2} = p\mu\sigma H \end{cases} \quad (14)$$

In the case of thin bars, considering the ratio η of magnetic fields determined above, the magnetic field on the two lateral sides of the bar could be approximated as follows:

$$H_{\mathrm{A}} \approx \frac{I}{(1+\eta)h}; \quad H_{\mathrm{B}} = \eta H_{\mathrm{A}}$$
$$E_0 = \frac{I}{\sigma b h} \approx \frac{1+\eta}{\sigma b} H_{\mathrm{A}}; \quad b << h \quad (15)$$

At the beginning of the process, when the current flows only on the surface of the conductors, the ratio η_0 must replace η in above equations.

Using the notations:

$$\nu = \sqrt{p\tau}; \quad \xi = \frac{x}{b}; \quad \tau = \mu\sigma b^2 \quad (16)$$

the solution of the equations (12) in conductor will be:

$$H(\xi,p) = \frac{H_{\mathrm{A}}}{p}\frac{\eta\sinh\nu\xi - \sinh\nu(1-\xi)}{\sinh\nu};$$
$$E(\xi,p) = \frac{H_{\mathrm{A}}}{\sigma b}\cdot\frac{\nu}{p}\cdot\frac{\eta\cosh\nu\xi + \cosh\nu(1-\xi)}{\sinh\nu} \quad (17)$$

Using the steady state electric field in conductor E_0, the Laplace transform of the transitory electric field will be:

$$E(\xi,p) = \frac{E_0}{1+\eta}\cdot\frac{\sqrt{\tau}}{\sqrt{p}}\cdot\frac{\eta\cosh\nu\xi + \cosh\nu(1-\xi)}{\sinh\nu} \quad (18)$$

The induction law applied to short-circuited in origin end line, gives the Laplace transform of the other end voltage:

$$U(z,p) = z\left[2E(0,p) + a\mu_0\, p\, H(0,p)\right] \quad (19)$$

where z is the length of the line.

The time constant τ of transitory electromagnetic field in 1 mm thickness copper or silver at 25 °C is ~ 75 μs.

4.4. Transitory parameters

The operational impedance of the bars at distance z from the short-circuited end results from (15)- (17):

$$Z(p) = \frac{1}{1+\eta}\left[r_{\mathrm{s}}\,\nu\frac{\eta+\cosh\nu}{\sinh\nu} + p\, l_{\mathrm{e}}\right] \quad (20)$$

where r_{s} is the direct current resistance of the bars and

$$r_s = \frac{2z}{\sigma b h}; \quad \theta = \frac{t}{\tau}; \quad l_{\mathrm{e}} = \mu_0\frac{a z}{h} \quad (21)$$

The Laplace transforms of the transitory resistance and inductance are defined as [2], [3]:

$$R(p) = \frac{Z(p)}{p} - l(0+)$$
$$L(p) = \frac{Z(p) - r_{\mathrm{s}}}{p^2} \quad (22)$$

From limit theorem of the Laplace transform it results

$$l(0+) = \lim_{p\to\infty}\left(p\,L(p)\right) = \frac{l_{\mathrm{e}}}{1+\eta} \quad (23)$$

Replacing (20) in (22) we obtain the Laplace transforms of the parameters:

$$R(p) = \frac{1}{1+\eta}\frac{r_{\mathrm{s}}}{p}\frac{\eta+\cosh\nu}{\sinh\nu}\nu \quad (24)$$

$$L(p) = \frac{1}{1+\eta}\left[\frac{l_e}{p} + \frac{r_s}{p}\frac{\eta + \cosh\nu - (1+\eta)\dfrac{\sinh\nu}{\nu}}{p\dfrac{\sinh\nu}{\nu}}\right] \quad (25)$$

The poles of the meromorphic function are the solution of the equation:

$$\sinh\nu = 0 \Rightarrow \nu_k = i\,k\,\pi \Rightarrow p_k = -\frac{(k\,\pi)^2}{\tau} \quad (26)$$

Applying the inversion theorem, it results for the transitory resistance of the line in Ω:

$$r(\theta) = r_s\left[1 + \frac{2}{1+\eta}\sum_{k=1}^{\infty}\left(1+(-1)^k\eta\right)e^{-(k\pi)^2\theta}\right] \quad (27)$$

and for the transitory inductance in H:

$$l(\theta) = \frac{1}{1+\eta}\left[l_e + \tau\, r_s\left(\frac{2-\eta}{6} - \frac{2}{\pi^2}\cdot\sum_{k=1}^{\infty}\frac{1+(-1)^k\eta}{k^2}e^{-(k\pi)^2\theta}\right)\right] \quad (28)$$

Taking into account (2) and that

$$r_s\tau = 2\,z\,\mu\frac{b}{h} \quad (29)$$

The inductance of z – length short-circuited end line in Henry become:

$$l(\theta) = \frac{\mu_0\, z}{(1+\eta)\,h}\left[a + 2\,\mu_r\, b\left(\frac{2-\eta}{6} - \frac{2}{\pi^2}\cdot\sum_{k=1}^{\infty}\frac{1+(-1)^k\eta}{k^2}e^{-(k\pi)^2\theta}\right)\right] \quad (30)$$

The inductance in direct current (at $t \to \infty$) is

$$l(\infty) = \frac{\mu_0\, z}{(1+\eta)\,h}\left[a + \mu_r\, b\,\frac{2-\eta}{3}\right]\ \text{[H]} \quad (31)$$

We will use the following functions (fig.7):

$$\Psi(\theta) = \frac{1}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}e^{-(k\pi)^2\theta}$$

$$\Psi_1(\theta) = \frac{-1}{\pi^2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}e^{-(k\pi)^2\theta} \quad (32)$$

The relative values of transitory parameters,

$$\rho(\theta) = \frac{r(t)}{r_s};\qquad \lambda(\theta) = \frac{l(\theta)}{l(\infty)} \quad (33)$$

can be written as

$$\rho(\theta) = \frac{r(t)}{r_s} = 1 - \frac{2}{1+\eta}\left[\Psi'(\theta) - \eta\,\Psi_1'(\theta)\right]$$

$$\lambda(\theta) = 1 - \frac{4\,\mu\, b\, z}{(1+\eta)\,h\,l(\infty)}\left[\Psi(\theta) - \eta\,\Psi_1(\theta)\right] \quad (34)$$

$$\lambda(\theta) = 1 - \frac{4}{\dfrac{a}{\mu_r b} + \dfrac{2-\eta}{3}}\left[\Psi(\theta) - \eta\,\Psi_1(\theta)\right]$$

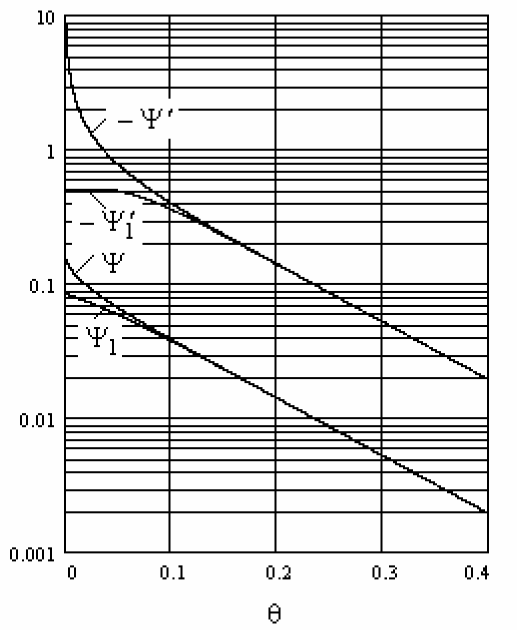

Fig. 7: Functions Ψ, Ψ_1 and its derivatives versus θ

The two transitory parameters satisfy the differential equation:

$$\rho(\theta) = 1 + K\frac{d\lambda}{d\theta};$$

$$K = \frac{1}{2(1+\eta)}\left[\frac{a}{\mu_r b} + \frac{2-\eta}{3}\right] \quad (35)$$

The line resistance is infinity at the beginning of current injection and become equal to the direct current resistance of the conductors at infinite time. The inductance is equal to external inductance at the beginning of the connection process (the magnetic field in conductor is zero) and increase to the final value (31), equal to the sum of external and internal inductance.

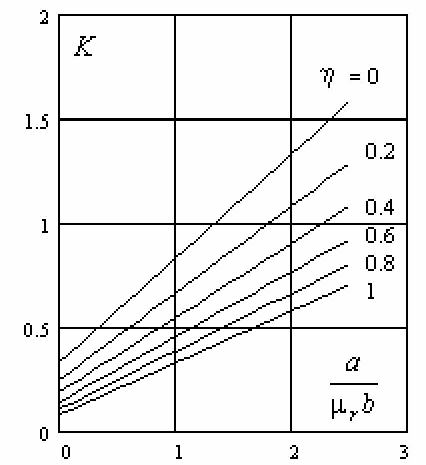

Fig. 8: Parameter *K* from differential eq. (33)

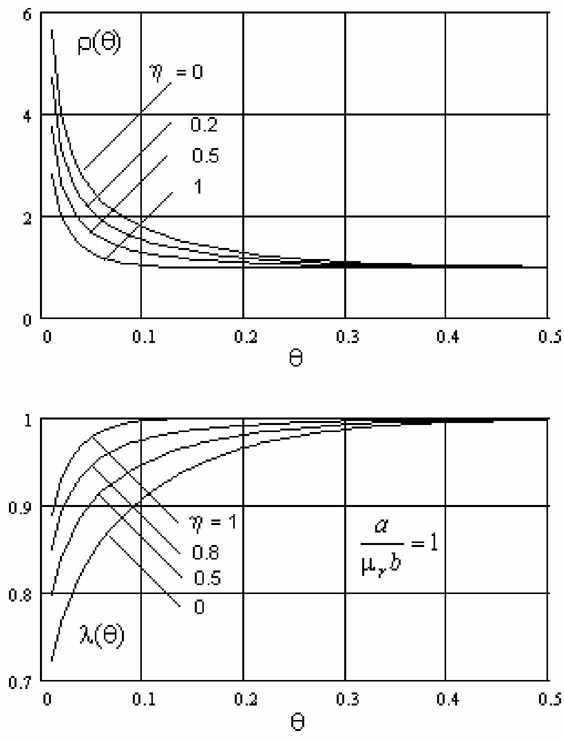

Fig. 9: Time variation of transitory parameters

From the previous figure we can see that in reality (in the case of finite height of the bars), due to the external magnetic field, the transitory parameters reach their stationary values faster than in the case of infinity high bars, when the external magnetic field is zero.

The inductance variation in fig 9 is given only for $a = \mu_r b$, for other values the last formula (34) must be used.

The equation (35) shows that the variation of the inductance is equivalent to a conductor resistance.

Conclusions

1. The design of the electrical fuse elements and their supports must take into consideration several effects, usually neglected as secondary, like the real distribution of the current in massive and large cross-section conductors. The conductor parameters and the thermal effects depends on the square power of the local current density and the dynamic effects depend also on the magnetic field distribution, which if function of the position of the back current conductors, in the vicinity of the fuse.
2. When the resistance of the variable section band is evaluated, the constriction resistance (3) must be added for each cross-section changing.
3. In the case of bent conductors or zigzag tapes the corner resistance (8) and the fig 4 has to be considered.
4. The electrodynamic forces acting on the fuse element depends on current density and the magnetic field. The local current densities for the mentioned cases can be determined using the equations (5) for the constriction or enlargement and (9) for the corners. The local magnetic field depends essentially on the position of the back current conductors, sometimes far away from the fuse.
5. The large cross-section conductors cannot be considered filiforms if the time constant τ is of the same order with the time constant of the considered processes. The real behavior of such conductors can be characterized by the transitory parameters, defined for step current injection or voltage step application. These parameters can be calculated using the equations (31), (34), (35) or evaluated from the fig. 9.

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